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DIAGNOSIS OF WAVE ACTIVITY OVER RAINBAND OF LANDFALL TYPHOON

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Abstract: A generalized wave-activity density, which is defined as an absolute value of production of three-dimensional vorticity vector perturbation and gradient of general potential temperature perturbation, is introduced and its wave-activity law is derived in Cartesian coordinates. Constructed in an agoestrophic and nonhydrostatic dynamical framework, the generalized wave-activity law may be applicable to diagnose mesoscale weather systems leading to heavy rainfall. The generalized wave-activity density and wave-activity flux divergence were calculated with the objective analysis data to investigate the character of wave activity over heavy-rainfall regions. The primary dynamical processes responsible for disturbance associated with heavy rainfall were also analyzed. It was shown that the generalized wave-activity density was closely correlated to the observed 6-h accumulative rainfall. This indicated that the wave activity or disturbance was evident over the frontal and landfall-typhoon heavy-rainfall regions in middle and lower troposphere. For the landfall-typhoon rainband, the portion of generalized wave-activity flux divergence, denoting the interaction between the basic-state cyclonic circulation of landfall typhoon and mesoscale waves, was the primary dynamic process responsible for the evolution of generalized wave-activity density.

Key words: dynamic meteorology; wave-activity density; wave-activity flux divergence; landfall typhoon, heavy-rainfall event

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1 INTRODUCTION

In atmospheric dynamics, physical fields are often decomposed into a basic-state portion and a perturbation portion. The interaction between basic state and disturbance is called wave-basic flow interaction. In the past, people frequently took zonal or temporal average of physical fields as their basic states. Thus, from this perspective, wave-basic flow interaction is also interpreted as interaction between wave and mean flow. As an important subject of atmospheric dynamics, the theories of wave-basic flow interaction may provide explanation to some important weather phenomena. For example, the upward propagation and breakdown theory of gravitational wave may explain the quasi-biennial oscillation between easterly and westerly of equatorial stratosphere[1]. The critical layer theory of planetary wave may explain the explosive warming of stratosphere[2]. The general baroclinic Eliassen-Palm flux theory may explain the acceleration and deceleration of upper-level jet stream in troposphere[3, 4].

The wave-basic flow interaction in nature implies two aspects of dynamic processes. The first aspect is the feedback effect of wave on basic-state flow which may be described by Eliassen-Palm flux theory[5]. The second aspect is the forcing of basic-state flow to wave which may be represented by "wave-activity" (conservation) law[6-11].

When Eliassen and Palm[12] investigated the transfer of mountain wave energy in basic flow varying with height, they firstly found the interaction between wave and mean flow, and then set up the notion of Eliassen-Palm flux (hereafter E-P flux)[12]. Since then,

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the E-P flux has been widely used as a tool for diagnosis of atmospheric general circulation in the context of transformed Eulerian mean zonal momentum equation, which is

$$\frac{\partial \overline{u}}{\partial t} - f\overline{v}^* = \frac{1}{\rho_0}\nabla \cdot \mathbf{F}^{ep}, \tag{1}$$

where $\overline{v}^*$ is the meridional component of residual circulation,

$$\mathbf{F}^{ep} = -\rho_0 \overline{u'v'}\vec{j} + \frac{f\rho_0}{N^2}\overline{v'\Phi'_z}\vec{k} \tag{2}$$

is the E-P flux and the sign " $\overline{\ }$ " denotes the zonal average. The physical meaning of E-P flux is clear from Eq. (1). The divergence of E-P flux ($\nabla \cdot \mathbf{F}^{ep} > 0$) accelerates the zonal mean westerly, while the convergence of E-P flux ($\nabla \cdot \mathbf{F}^{ep} < 0$) decelerates the zonal mean westerly. For plane wave in slowly varying basic flow, with the Wentzel-Kramers-Brillouin (WKB) approximation, the E-P flux is parallel to the group velocity vector of plane wave, representing the direction of wave-energy propagation, that is

$$\mathbf{F}^{ep} = A\mathbf{c}_g, \tag{3}$$

where A is the wave-activity density and $\mathbf{c}_g$ is the group velocity vector.

Since the E-P flux theory was firstly proposed, it has been generalized to various forms. In the investigation of a new generalization of the Eliassen-Palm relation, Andrews and McIntyre[13] found that the ill-conditioned behavior of term in $\overline{u'v'}$ made it difficult to exactly calculate $\frac{\partial \overline{u}}{\partial t}$. To avoid the problem, they introduced residual mean meridional circulation successfully into the Eulerian transformed mean zonal momentum equation to generalize the E-P flux theory[13]. However, their theory was constructed in barotropic environment and was not applicable to baroclinic atmosphere. In addition, the particle displacement was required by their generalized E-P flux theory, which obstructed its application in practice. Gao et al.[3] extended the results of Andrews and McIntyre[13] by invoking scale comparison of magnitudes and Gardner-Morikawa transformation. Since the atmospheric stratification was involved, the extension of E-P flux theory by Gao et al.[3] might be used to diagnose weather phenomenon over jet stream and front in baroclinic atmosphere. It was realistic representation that the E-P flux was creatively applied to the acceleration of upper-level jet stream. This was an important application of E-P flux theory. Besides this, their theory did not require the information of particle displacement, which promoted the wider utility of E-P flux theory in practice. In a word, the two limitations of Andrews and McIntyre[13] disappeared in the theory of Gao et al.[3]. Considering the baroclinicity effect, Gao et al.[14] further enriched and improved their E-P flux theory by invoking the meridional gradient of density[14]. Their analysis revealed that the acceleration of upper-level jet stream under consideration of baroclinicity and stratification effects was larger than that considering stratification effect only.

The "wave activity" or "wave activity density" was defined as a disturbance quantity which is quadric or higher order in disturbance amplitude in the small-amplitude limit by Scinocca and Shepherd[15] and Haynes [16]. The wave-activity density generally satisfies a general form of wave-activity law

$$\frac{\partial A}{\partial t} + \nabla \cdot \mathbf{F} = S, \tag{4}$$

where A is the wave-activity density, $\mathbf{F}$ is the wave-activity flux, and S is the source and sink of wave-activity density. Since both A and $\mathbf{F}$ are the terms quadric or higher-order in disturbance fields, and A represents a kind of wave energy, the wave-activity law may describe the evolution of disturbance. It can be seen from the above wave-activity law that the convergence or divergence of wave-activity flux may cause the local congregation or dissipation of wave-activity density and further lead to the development or attenuation of transient wave. For a closed system with no inflow and outflow on boundary, and without source or sink, the wave activity is conservational, namely, $\frac{d}{dt}\iiint Adxdydz = 0$. In the past decades, people have carried out comprehensive investigation of wave-basic flow interaction and constructed all kinds of wave-activity laws[17-24].

The most of previous wave-activity theories are generally constructed in pressure coordinates with quasi-geostrophic approximation or in isentropic coordinates. For example, based on the quasi-geostrophic potential vorticity equation, McIntyre and Shepherd[25] employed an "Energy-Casimir" method to construct a conservation relation for disturbances to parallel and nonparallel basic flows. Haynes[16] took into account of the effects of diabatic heating and turbulent dissipation to investigate the finite-amplitude, local wave-activity relations for disturbances to zonal and nonzonal basic flows in isentropic coordinates by using the extensions of Momentum-Casimir and Energy-Casimir methods[16]. Since these wave-activity theories naturally are hydrostatic, they are suitable for large-scale weather systems, and cannot be applied to the nonhydrostatic and ageostrophic mesoscale weather systems. Scinocca and Shepherd[15] derived a nonhydrostatic and ageostrophic wave-activity conservation laws for finite amplitude from a Hamiltonian system. However, their theories were confined to a two-dimensional system. Realizing the above questions in the previous investigation of wave activity, Ran and Gao[26] derived a three-dimensional, nonhydrostatic and ageostrophic local wave-activity law for pseudomomentum from primitive equations in Cartesian coordinates by using an extension of momentum–Casimir method. The wave-activity density and wave-activity flux in the local wave-activity law for pseudomomentum were expressed entirely in terms of Eulerian quantities so that they were easily calculated with atmospheric grid data and did not require knowledge of particle distance. Constructed in nonhydrostatic and ageostrophic dynamical framework, the local wave-activity law is applicable to diagnosing the evolution and propagation of mesoscale weather systems. This form of wave-activity law was constructed for the first time and the result may be, to some extent, regarded as a supplement to the theory of Haynes.

The choice of basic state is a key to wave-activity law. Generally, basic states are assumed to be stationary. This means that there is no feedback effect of wave on basic flow and the two aspects of wave-basic flow interaction are separated. Knowing the insufficiency and shortcoming of wave-activity law, Ran and Boyd[27] further constructed a nonhydrostatic and ageostrophic interaction equation from the zonally symmetric primitive equations in Cartesian coordinates by using the Momentum-Casimir method. The equation may be employed to investigate the interaction between nonhydrostatic and ageostrophic mesoscale transient wave and nonstationary and nonconservative basic flow. The work of Ran and Boyd[27] was characterized by linking the two dynamic aspects of wave-flow interaction.

The Energy-Casimir method is one of the important methods to construct wave-activity law in previous investigation. A Casimir invariant—defined as a single-value function of some conservative quantities—is commonly introduced in the derivation with Momentum- and Energy-Casimir methods. Since the Casimir invariant generally does not have an explicit and specific expression, it is hard to calculate wave-activity densities. This may obstruct these theories from application in practice. Furthermore, the previous studies implicitly assumed that the atmosphere was dry and excluded the effect of moisture. In fact, the real atmosphere is not absolutely dry. If precipitation occurs, moist atmosphere is saturated and cloud hydrometeors play an important role in development of precipitable weather system. Thus, the effect of water vapor on disturbance should not be ignored in dynamics. Consequently, Gao and Ran[28] consider the effect of water vapor to develop three-dimensional nonhydrostatic and ageostrophic general moist wave-activity laws in Cartesian coordinates with the aid of potential vorticity theorem. Since their work did not contain the Casimir invariant, their theory was easily carried out in practice. Their analysis showed that the three specific wave-activity densities were closely related to the simulated rain rate, which suggests that the wave-activity densities might serve as track for detecting precipitation. These investigations enrich the theory of wave-basic flow interaction and remarkably extend it to the field of mesoscale dynamics.

The purpose of this paper is to generalize the result of Gao and Ran[28] and diagnose the character of wave activity over rainband of landfall typhoon. The wave-activity law for disturbance to stationary basic states is generalized in the next section. The application of the generalized wave-activity law to a landfall typhoon associated with heavy rainfall is conducted in section 3. The conclusion is given in section 4.

2 THE GENERALIZED WAVE-ACTIVITY LAW

For an inviscid, compressible, diabatic, nonhydrostatic and ageostrophic flow, the governing equations on the f plane in Cartesian coordinates (x, y, z) may be given by

$$\frac{du}{dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{5}$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{6}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g, \tag{7}$$

$$\frac{d\rho}{dt} + \rho\nabla\cdot\mathbf{v} = 0, \tag{8}$$

$$\frac{d\theta}{dt} = S_\theta, \tag{9}$$

$$\frac{dq_v}{dt} = S_{q_v}, \tag{10}$$

$$p = \rho RT(1 + \lambda q_v), \tag{11}$$

where $\mathbf{v} = (u,\ v,\ w)$ is the three-dimensional velocity vector and the other signs are common in meteorology. The general potential temperature is introduced[29]

$$\theta^* = \theta\exp\left[\frac{L_v q_{vs}}{c_p T_c}\left(\frac{q_v}{q_{vs}}\right)^k\right] \tag{12}$$

where q_{vs} is the saturated specific humidity, T_c is the temperature at condensation lift level, and k is an experiential constant. The equation of θ^* can be expressed as

$$\frac{d\theta^*}{dt} = S_{\theta^*}, \tag{13}$$

where

$$S_{\theta^*} = \theta^* S_\theta/\theta + \theta^* \ln(\theta^*/\theta)[k\, d(\ln q_v)/dt + (1-k)d(\ln q_{vs})/dt - d(\ln T_c)/dt]$$

is the source or sink of θ^*.

All physical fields are divided into a basic-state part (denoted by the subscript "$_0$") and a perturbation part (denoted by the subscript "$_e$"), namely

$$\begin{aligned}
u &= u_0(x,y,z) + u_e(x,y,z,t),\\
v &= v_0(x,y,z) + v_e(x,y,z,t),\\
w &= w_0(x,y,z) + w_e(x,y,z,t),\\
p &= p_0(x,y,z) + p_e(x,y,z,t),\\
\rho &= \rho_0(x,y,z) + \rho_e(x,y,z,t),\\
T &= T_0(x,y,z) + T_e(x,y,z,t),\\
\theta &= \theta_0(x,y,z) + \theta_e(x,y,z,t),\\
\theta^* &= \theta_0^*(x,y,z) + \theta_e^*(x,y,z,t),\\
q_v &= q_{v0}(x,y,z) + q_{ve}(x,y,z,t),\\
q &= q_0(x,y,z) + q_e(x,y,z,t).
\end{aligned} \tag{14}$$

As a set of steady solution to the governing equations, the basic states satisfy

$$\mathbf{v}_0 \cdot \nabla u_0 - f v_0 = -\frac{1}{\rho_0}\frac{\partial p_0}{\partial x}, \tag{15}$$

$$\mathbf{v}_0 \cdot \nabla v_0 + f u_0 = -\frac{1}{\rho_0}\frac{\partial p_0}{\partial y}, \tag{16}$$

$$\mathbf{v}_0 \nabla w_0 = -\frac{1}{\rho_0}\frac{\partial p_0}{\partial z} - g, \tag{17}$$

$$\mathbf{v}_0 \cdot \nabla \theta_0^* = 0, \tag{18}$$

and the linear perturbation equations are

$$\frac{\partial u_e}{\partial t} = -\mathbf{v}_e \cdot \nabla u_0 - \mathbf{v}_0 \cdot \nabla u_e + f v_e - \frac{1}{\rho_0}\frac{\partial p_e}{\partial x} + \frac{\rho_e}{\rho_0^2}\frac{\partial p_0}{\partial x}, \tag{19}$$

$$\frac{\partial v_e}{\partial t} = -\mathbf{v}_e \cdot \nabla v_0 - \mathbf{v}_0 \cdot \nabla v_e - f u_e - \frac{1}{\rho_0}\frac{\partial p_e}{\partial y} + \frac{\rho_e}{\rho_0^2}\frac{\partial p_0}{\partial y}, \tag{20}$$

$$\frac{\partial w_e}{\partial t} = -\mathbf{v}_e \cdot \nabla w_0 - \mathbf{v}_0 \cdot \nabla w_e - \frac{1}{\rho_0}\frac{\partial p_e}{\partial z} + \frac{\rho_e}{\rho_0^2}\frac{\partial p_0}{\partial z}, \tag{21}$$

$$\frac{\partial \theta_e^*}{\partial t} = -\mathbf{v}_e \cdot \nabla \theta_0^* - \mathbf{v}_0 \cdot \nabla \theta_e^* + S_{\theta^*}, \tag{22}$$

where $\mathbf{v}_e = (u_e,\ v_e,\ w_e)$ is the velocity vector perturbation, and $\mathbf{v}_0 = (u_0,\ v_0,\ w_0)$ is the basic-state velocity vector.

We consider a perturbation scalar

$$|J| = \left|\boldsymbol{\omega}_e \cdot \nabla \theta_e^*\right|, \tag{23}$$

in which $\boldsymbol{\omega}_e = \nabla \times \mathbf{v}_e$ is the perturbation vorticity. Because $|J|$ is quadric-order in disturbance fields and always is definitely positive, $|J|$ is capable of describing the energy of disturbance, indicating the strength of disturbance.

Taking a partial derivative of Eq. (23) with regard to time, one may obtain

$$\frac{\partial |J|}{\partial t} = \nabla \cdot \left[\frac{J}{|J|}\left(\boldsymbol{\omega}_e \frac{\partial \theta_e^*}{\partial t} + \frac{\partial \boldsymbol{\omega}_e}{\partial t}\theta_e^*\right)\right]. \tag{24}$$

After eliminating the local changes of perturbation fields in Eq. (24) using the linear perturbation equations (19) – (22), one may express Eq. (24) in a flux form

$$\frac{\partial |J|}{\partial t} + \nabla \cdot \mathbf{F} = \nabla \cdot \left(\boldsymbol{\omega}_e S_{\theta_e^*}\right), \tag{25}$$

where $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$ is wave-activity flux whose components are given by

$$\mathbf{F}_1 = \mathbf{v}_0 |J|, \tag{26}$$

$$\mathbf{F}_2 = \frac{J}{|J|}\left[\mathbf{v}_e\left(\boldsymbol{\omega}_0 \cdot \nabla \theta_e^*\right) - \boldsymbol{\omega}_0\left(\mathbf{v}_e \cdot \nabla \theta_e^*\right)\right], \tag{27}$$

$$\mathbf{F}_3 = \frac{J}{|J|}\boldsymbol{\omega}_e\left(\mathbf{v}_e \cdot \nabla \theta_0^*\right), \tag{28}$$

$$\mathbf{F}_4 = \frac{J}{|J|}\left(\nabla p_0 \times \nabla \frac{\rho_e}{\rho_0^2} - \nabla p_e \times \nabla \frac{1}{\rho_0}\right)\theta_e^*. \tag{29}$$

Because Eq. (25) is expressed in a flux form and both $|J|$ and $\mathbf{F}$ are quadric in disturbance fields, Eq. (25) may be interpreted as a generalized wave-activity law for the general wave-activity density $|J|$ and the wave-activity flux $\mathbf{F}$. The term on the right-hand side of Eq. (25) is referred to as the source or sink of wave-activity density.

The wave-activity flux $\mathbf{F}$ is the sum of four portions. The first portion $\mathbf{F}_1$ represents the transportation of wave-activity density by basic-state velocity. The second portion $\mathbf{F}_2$ denotes the covariance between the transportation of linear potential vorticity substance by perturbation velocity vector and the interaction between basis-state vorticity and advection of perturbation general potential temperature by perturbation velocity vector. The third portion $\mathbf{F}_3$ denotes the interaction between perturbation vorticity and advection of basic-state general potential temperature by perturbation velocity vector. The fourth portion $\mathbf{F}_4$ represents the covariance between linear perturbation solenoid and general potential temperature perturbation.

In the following section, the generalized wave-activity law obtained here will be used to diagnose the wave activity character over heavy-rainfall regions associated with front and landfall typhoon.

3 CASE STUDY

There were two heavy-rainfall events concurrently taking place during the period of 00 UTC 11 – 00 UTC 15 August 2004. One event was associated with a cold front in the north of China and the other was associated with the landfall typhoon "Rananim" that occurred in the southeast of China. The disturbance (namely, wave activity) associated with the two observed weather systems will be analyzed with the help of the above wave-activity law in this section. The objective analysis data, which was produced by ADAS [ARPS (Advanced Regional Prediction System) Data Analysis System], was employed to calculate the wave-activity density and wave-activity flux divergence in Eq. (25). Due to the limitation of the analysis data, the local change and source or sink of wave-activity density in Eq. (25) were excluded from the calculation.

In order to analyze the total characteristics of disturbance represented by wave-activity density in heavy-rainfall events, we calculated the vertical integration of $|J|$ instead of its value at some single level. The vertical integration denoted by $\langle \bullet \rangle = \int_{z_s}^{z_t} \bullet dz$ was carried out from the bottom of ARPS model $z_s = 250\text{m}$ to the top of ARPS model $z_t = 17750\text{m}$.

As shown in Fig. 1, there existed one observed heavy-rainfall region in the north of China at 00 UTC 12 August 2004. This heavy rainfall is clearly a type of frontal precipitation and took a band structure stretching northeastwards from the west of Shanxi province (40°N, 102°E) to the south of Jilin province (45°N, 126°E). At the same time, a local weak rainfall region mainly occurred in the southeast of Zhejiang province (29°N, 121°E), which resulted from the landfall typhoon "Rananim". The northern frontal rainband was covered by a high-value area of wave-activity density. Since the main body of typhoon stayed in the sea and its small portion landed, the positive center of wave-activity density matching with the typhoon was off land and its northwest edge overlapped the local weak rainfall region in the southeast of China. After that, the frontal rainband moved southeastwards and always was accompanied by the high-value wave-activity density. As the typhoon gradually landed, its surface rainfall increased and was located in the main body of wave-activity density. At 12 UTC 12, the wave-activity density showed the same horizontal distribution patterns as the two heavy-rainfall regions and overlapped them. The anomaly of wave-activity density implied that the dynamic and thermodynamic properties over heavy-rainfall regions were violently disturbed. The southeastern landfall-typhoon rainfall was much larger than the northern frontal rainband, and the wave-activity density over the latter was much weaker than that over the former. The explanation to this may be reduced to two points. The first point is that the thermodynamic property of typhoon gives rise to the fact that the surface of general potential temperature perturbation in landfall typhoon is steeper than that in front. Thus the horizontal gradient of general potential temperature in landfall typhoon is more remarkable. The second point is that the cyclonic vorticity perturbation of landfall typhoon is much stronger that of front. The northern frontal rainband was broken down into two parts and the landfall-typhoon rainfall stayed in the former place at 18 UTC 12. Both of them matched with the high-value area of wave-activity density. The northern frontal rainband was gradually weakened until it disappeared since then. The landfall-typhoon rainfall moved westwards and decreased slowly. The wave-activity density was kept anomalous over the two rainfall regions and was negligible in the no-rainfall regions. As the heavy-rainfall events decayed, the wave-activity density became feeble. For example, the wave-activity density in the north of China almost disappeared at 06 UTC 13 when the northern frontal precipitation finished. At 00 UTC 14 the landfall-typhoon rainfall region arriving at the domain of (27°N – 32°N, 113°E – 118°E) with a center at (29°N, 115°E) was decomposed and began to decline gradually. In the meantime, the wave-activity density laying over the landfall-typhoon precipitation region was also lessened. It is clear that during the frontal and landfall-typhoon heavy-rainfall events the wave-activity density collocated with the observed surface rainfall region and its horizontal scope was wider than the organized large-scale heavy-rainfall region. Although there did not exist distinct wave-activity density in some isolated local rainfall areas, the wave-activity density on the

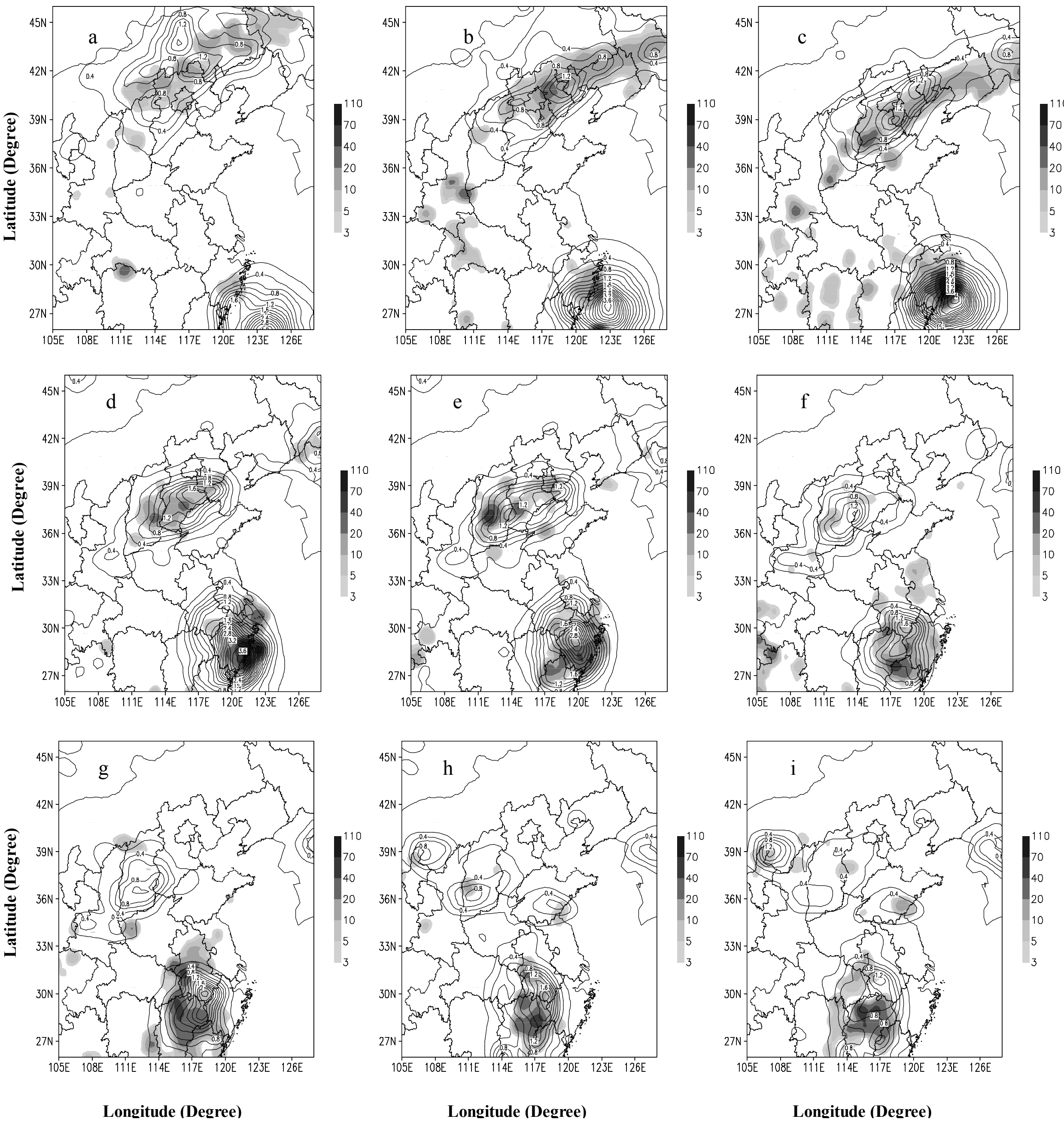


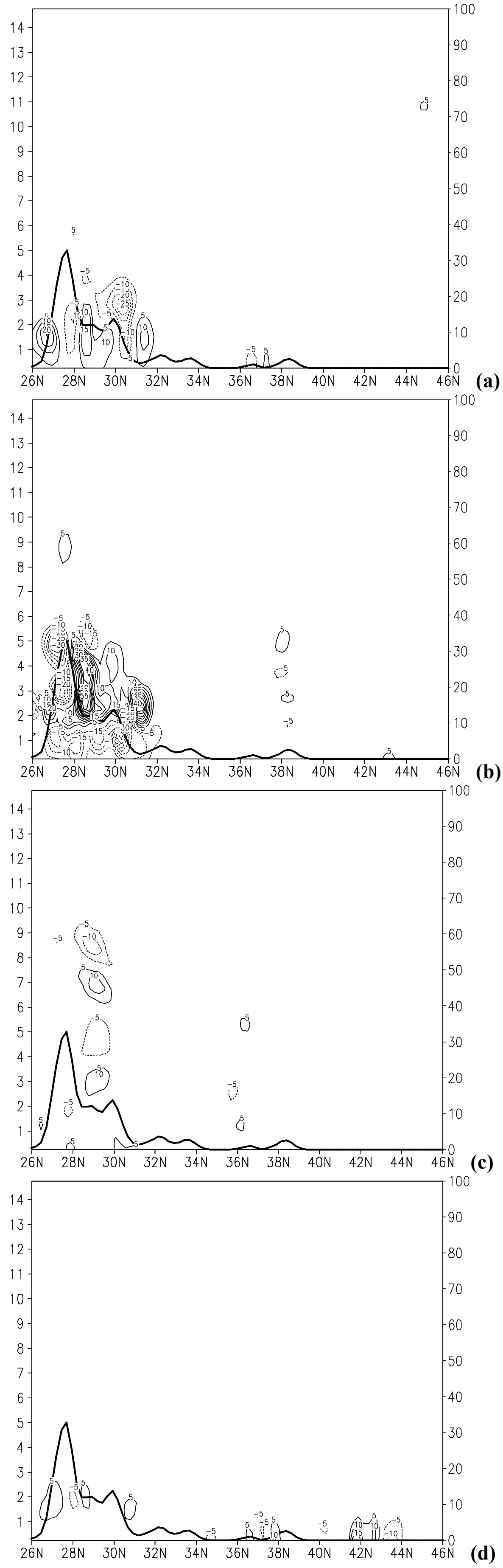
Fig. 1 Horizontal distributions of wave-activity density vertically integrated at 00 UTC 12 (a), 06 UTC 12 (b), 12 UTC 12 (c), 18 UTC 12 (d), 00 UTC 13 (e), 06 UTC 13 (f), 12 UTC 13 (g), 18 UTC 13 (h) and 00 UTC 14 (i) August 2004 (Unit: 10^{-3} K s^{-1}). The gray shade denotes the observation of 6-h accumulated surface rainfall. (Unit: mm)

whole was consistent with the observed rainfall in the horizontal distributions.

It also should be emphasized that in some isolated and local heavy-rainfall areas, the wave-activity density did not present a strong signal. The explanation to this is focused on two points. One point is that the analysis data used here is too coarse to contain more valuable information of mesoscale and small scale local weather systems engendering to the isolated and local heavy-rainfall. Another important point is due to the fact that the precipitation is a result of very complicated cloud microphysical processes while the wave-activity density describes a macroscopically dynamical process. There is not a straightforward relation between the two kinds of physical processes. They are connected together in virtual of a third party. Thus, although the wave-activity density may share a similar distribution pattern and evolution tendency with the observed rainfall, they are not identical to each other. Consequently, we may infer that the wave-activity density can more efficiently portray the organized and systemic weather disturbance than the isolated and local disturbance.

The above diagnosis manifests that the wave-activity density shares similar horizontal distribution patterns and temporal evolution trends with the observation of 6-h accumulative rainfall. This indicates that the wave-activity density is closely related to the observed rainfall. The consistence between wave-activity density and surface rainfall suggests that there exists an evident wave (namely, disturbance) activity over the frontal and landfall-typhoon heavy-rainfall regions. In other words, the wave presented by wave-activity density is the dominative weather system resulting in heavy rainfall. In addition, the landfall-typhoon rainfall matches with a more intensive wave-activity density than the frontal rainfall, implying that the wave over the former is more active than that over the latter on the whole.

In order to analyze the chief dynamic physical processes responsible for the variation of wave-activity density, we utilized the analysis data to calculate the four portions of wave-activity flux divergence on the left-hand side of Eq. (25). Due to the limitation of the analysis data, the two terms of local change of wave-activity density on the left-hand side of Eq. (25) and source or sink on the right-hand side of Eq. (25) were excluded from calculation. Thus, the thermodynamic processes and cloud microphysical processes answerable for wave-activity density were not taken into account in this paper. The flux divergences of Eqs. (26) – (29) in vertical-meridional cross section along the landfall-typhoon rainfall centers were presented in Fig. 2. As it was shown, over the landfall-typhoon rainband about 26°N – 31°N, the term $\nabla \cdot \mathbf{F}_2$ exerted dominative influence in the middle and lower troposphere. The other three terms, $\nabla \cdot \mathbf{F}_1$, $\nabla \cdot \mathbf{F}_3$ and $\nabla \cdot \mathbf{F}_4$, made lesser contributions to the wave-activity density. The low pressure of landfall typhoon was accompanied with the cyclonic circulation. So its dynamic property was characterized by intensive vertical vorticity. Accordingly, the basic-state vertical vorticity of landfall typhoon was much prominent than that near the front. This led to that the term $\nabla \cdot \mathbf{F}_2$ of landfall typhoon associated with basic-state vorticity was more primary than the other three terms. On the other hand, the importance of $\nabla \cdot \mathbf{F}_2$ suggested that the interaction between the basic-state cyclonic circulation of landfall typhoon and mesoscale disturbance dominated the variation of mesoscale waves embedded in landfall typhoon.



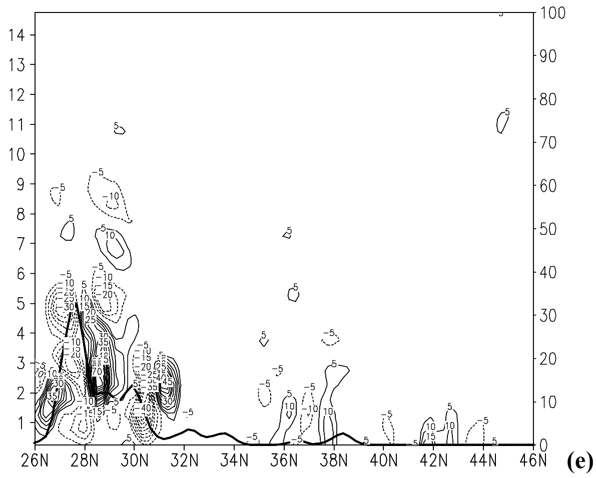


Fig.2 Vertical cross sections of $\nabla \cdot \mathbf{F}_1$ (a), $\nabla \cdot \mathbf{F}_2$ (b), $\nabla \cdot \mathbf{F}_3$ (c), $\nabla \cdot \mathbf{F}_4$ (d) and $\nabla \cdot \mathbf{F}$ (e, Unit: 10^{-12} K m^{-1} s^{-2}) zonally averaged over the longitude belt of 116 – 119°E at 06 UTC 13 August 2004. Thick solid line: the observation of 6-h accumulated surface rainfall zonally averaged over the longitude belt of 116 – 119°E (Unit: mm). The abscissa is for the latitude and the right ordinate is for rainfall. (unit: mm)

4 DISCUSSION AND CONCLUSION

In this paper, based on the potential vorticity theorem, we introduce the production of three-dimensional vorticity vector perturbation and gradient of general potential temperature perturbation. The absolute value of the production is defined as wave-activity density and the associated wave-activity law is derived in Cartesian coordinates. This work is the generalization version of Gao and Ran[28]. As a kind of perturbation energy, the wave-activity density is definitively positive in sign so that it is capable of representing the evolution of disturbance. Constructed in the three-dimensional agoestrophic and non-hydrostatic dynamical framework, the wave-activity law may be applicable to mesoscale weather systems often leading to precipitation. Unlike momentum-Casimir and energy-Casimir methods, the method used here does not involve Casimir function so that the wave-activity law is easily calculated with Eulerian grid-point data.

In order to analyze the wave activity over frontal and landfall-typhoon rainfall regions in heavy-rainfall events, the wave-activity density was calculated by using the grid analysis data. The relationship between it and the observation of 6-h accumulative surface rainfall was investigated. At the same time, to examine the primary dynamic process responsible for the evolution of wave (described by wave-activity density), the four portions of wave-activity flux divergence were also calculated.

The result revealed that the wave-activity density was basically consistent with the observation of 6-h accumulative surface rainfall in the horizontal distribution patterns and temporal evolution trends. Their consistence suggests that there exists an evident wave (namely, disturbance) activity over the frontal and landfall-typhoon heavy-rainfall regions. The wave may be referred to as the dominative weather system resulting in heavy rainfall. Furthermore, the wave over the landfall-typhoon heavy-rainfall region on the whole was more active than that over the frontal heavy-rainfall region.

The analysis of wave-activity flux divergence manifested that for the landfall-typhoon rainband, the most contribution of wave-activity flux divergence was given rise to by the portion denoting the interaction between the basic-state cyclonic circulation of landfall typhoon and mesoscale waves embedded in landfall typhoon.

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