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# **THE EFFECTIVENESS OF GENETIC ALGORITHM IN CAPTURING CONDITIONAL NONLINEAR OPTIMAL PERTURBATION WITH PARAMETERIZATION "ON-OFF" SWITCHES INCLUDED BY A MODEL**

FANG Chang-luan (方昌銮) $^{1,2}$ , ZHENG Qin (郑 琴) $^{1}$ 

(1. Institute of Science, PLA University of Science and Technology, Nanjing 211101; 2. Oceanic Hydrometeorological Center of the South Sea Navy Fleet, Zhanjiang 524001)

**Abstract:** In the typhoon adaptive observation based on conditional nonlinear optimal perturbation (CNOP), the 'on-off' switch caused by moist physical parameterization in prediction models prevents the conventional adjoint method from providing correct gradient during the optimization process. To address this problem, the capture of CNOP, when the "on-off" switches are included in models, is treated as non-smooth optimization in this study, and the genetic algorithm (GA) is introduced. After detailed algorithm procedures are formulated using an idealized model with parameterization "on-off" switches in the forcing term, the impacts of "on-off" switches on the capture of CNOP are analyzed, and three numerical experiments are conducted to check the effectiveness of GA in capturing CNOP and to analyze the impacts of different initial populations on the optimization result. The result shows that GA is competent for the capture of CNOP in the context of the idealized model with parameterization 'on-off' switches in this study. Finally, the advantages and disadvantages of GA in capturing CNOP are analyzed in detail.

**Key words:** dynamic meteorology; typhoon adaptive observation; genetic algorithm; conditional nonlinear optimal perturbation; switches; moist physical parameterization

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## **1 INTRODUCTION**

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Among the scientific investigations on the atmosphere and ocean, many physical phenomena can be viewed as perturbations added to the basic flow and the evolution of physical phenomena come down to the investigation of perturbations' evolution in mathematics. So the determination of the fastest growing initial perturbations is of central importance and has been an attractive issue since the work of Lorenz  $\left[1\right]$ . The linear approach for capturing the fastest growing initial perturbation is widely adopted with the assumption that the initial perturbation is sufficiently small such that its evolution can be governed approximately by the tangent linear model (TLM) of a nonlinear model. Then the calculation of linear fastest growing perturbation is reduced to the evaluation of the linear singular vector (LSV) and linear singular value (LSVA). However, the

motions of the atmosphere and ocean are governed by complicated nonlinear systems. In order to study the nonlinear mechanism of the amplification of initial perturbations, Mu  $^{[2, 3]}$  proposed the concept of conditional nonlinear optimal perturbation (CNOP). CNOP is the initial perturbation whose nonlinear evolution attains the maximal (or "optimal") value of the cost function constructed according to the physical problems of interest at a specified time with physical constraint conditions.

Presently CNOP has already been used in ENSO predictability  $[4]$ , ensemble prediction  $[5]$ , spring predictability barrier for El Niño events<sup>[6,7]</sup>, nonlinear characteristics of El Niño events <sup>[8]</sup>, El Niño and La Niña amplitude asymmetry <sup>[9]</sup>, stability, sensitivity and predictability [10] and so on, but most of the models governing the nonlinear evolution of initial perturbation are simplified ones with continuous physical variables.

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**Biography:** FANG Chang-luan, M.S. candidate, mainly undertaking the study on nonlinear optimization and data assimilation.

E-mail for correspondence author: [clfang\\_pla@sohu.com](mailto:clfang_pla@sohu.com)

Recently, CNOP is used in typhoon adaptive observation with the MM5 model  $\left[$ <sup>[11]</sup>. When moist physical parameterizations are included, however, the cost function, which describes the conditional nonlinear growth of initial perturbation, is filled with discontinuities induced by the moist physical parameterization. At times some terms of governing equation(s) are not differential (even discontinuous) with respect to time (model variables) at some critical points, and this phenomenon is generally called the "on-off" switch. Moist physical parameterizations in atmospheric numerical models may trigger "on-off" switches  $[12 - 17]$ .

In the aforementioned CNOP related literature <sup>[2-1</sup>] <sup>11]</sup>, some information gradient-based optimization algorithms are adopted for the optimization process, and the conventional adjoint method is used to provide the gradient of cost function for the optimization process. However, a lot of investigations have revealed that the effectiveness of the conventional adjoint method is confronted with great challenges when there are discontinuous switches in the governing equation  $(Xu^{[12,13]}; Zou^{[14]}; Mu^{[15,16]}; Zheng^{[17]})$ . So, when full physical parameterizations are adopted in typhoon adaptive observation and typhoon- and storm-precipitation-related sensitivity analyses, the conventional adjoint method fails to provide the correct gradient.

If we continue to use the idea of non-smooth optimization ( $Zhu^{[18]}$ ) to consider the problem of moist physical parameterization with CNOP in governing equation(s), the genetic algorithm (GA) can be used to replace gradient or sub-gradient based methods, and then the impact of "on-off" switches on the optimization process may be decreased or eliminated. GA is based on the simulation of human being's and biological evolution to search an optimal, general solution, it starts with a set of (rather than one) solutions and a number of simple genetic operators are used during the iteration processes to improve current solutions, which are evaluated only by the value of their cost function. The whole iteration processes are carried through without any gradient (or sub-gradient) related information.

There are at least two potential advantages to use GA to capture CNOP with moist physical parameterizations in typhoon adaptive observation, typhoon- and storm-precipitation-related sensitivity analyses. Firstly, no adjoint model is required in GA and the model used in the optimization process can be exactly consistent with the prediction model, and thereby, the results of the above adaptive observations and sensitivity analyses can reflect the exact characteristics of the prediction model. Secondly, parallel procedures can be easily designed in GA for its

inherent parallel computation characteristics, which thereby can take full advantage of the rapidly developing computational parallel technology to improve the actual effect of related investigation.

This study aims at checking the effectiveness of GA in capturing CNOP with the "on-off" switches in the governing equation by using an idealized model of a partial differential equation with the parameterization "on-off" switch in the forcing term to reflect the impact of discontinuity, which is induced by the switch, on the performance of the conventional adjoint method in the process of capturing CNOP. The descriptions of the model and CNOP are presented in Section 2, and the procedure of GA in capturing CNOP is formulated in detail in Section 3. Section 4 gives the analyses of the failure of the conventional adjoint method in capturing CNOP with the "on-off" switch and numerical experiments are conducted to check the effectiveness of GA in capturing CNOP and the impacts of different initial population on the optimization results, and finally the advantages and disadvantages of GA in capturing CNOP are analyzed in Section 5.

### **2 DESCRIPTIONS OF MODEL AND CNOP**

### 2.1 *Model*

As shown in Mu  $\left[16\right]$ , the authors proposed the following partial differential governing equation with a parameterized "on-off" switch in the source term, i.e.,

$$
\begin{cases}\n\frac{\partial q}{\partial t} + a \frac{\partial q}{\partial l} = F - gH(q - q_c), & 0 \le l \le L, 0 \le t \le T \\
q(t, l)|_{t=0} = q_0(l), & 0 \le l \le L \\
\frac{\partial q(t, l)}{\partial l}|_{t=0} = 0, & 0 \le t \le T.\n\end{cases}
$$
\n(1)

This equation describes the evolution of specific humidity *q* along one grid line *l*, where  $q(t, l) \ge 0$ denotes the specific humidity,  $q_c$  is the saturation specific humidity (namely threshold); *t* is the time; *l* stands for either horizontal variable *x* (or *y* ) or vertical variable *z* . For more detailed description, refer to Mu<sup>[16]</sup>. In order to reflect the switch problem in the real model where the "on-off" switch can be triggered repeatedly, the constant  $F$  is converted to a function  $F(t)$ , and  $q_0(l) < q_c$  and  $F(0) - g > 0$  are set to make sure that the "on-off" switch can be triggered repeatedly in some space gridpoints within  $[0, T]$ . So the governing equation is rewritten as

$$
\begin{cases}\n\frac{\partial q}{\partial t} + a \frac{\partial q}{\partial t} = F(t) - gH(q - q_c), & 0 \le l \le L, 0 \le t \le T \\
q(t, l)|_{t=0} = q_0(l), & 0 \le l \le L \\
\frac{\partial q(t, l)}{\partial l}|_{t=0} = 0, & 0 \le t \le T,\n\end{cases} \tag{2}
$$

where  $F(t) = a - bt$ ,  $a = 8$ ,  $b = 11$ ,  $g = 7$ ,  $q_c = 0.58$ ,

 $q_0 (l) = 0.28 - 0.15 \sin (pl/2)$ , and Eq.(2) is discretized as follows:

$$
q_0^i = q_0(l_i), i = 0, 1, \cdots, M
$$
 (3)

$$
q_k^0 = q_{k-1}^0 + \left[ F(t_{k-1}) - gH(q_{k-1}^0 - q_c) \right] \Delta t, 1 \le k \le N \tag{4}
$$

$$
q_{k}^{i} = q_{k-1}^{i} - \frac{\Delta t}{\Delta l} a(t_{k-1}, l_{i}) (q_{k-1}^{i} - q_{k-1}^{i-1})
$$
  
+ 
$$
\left[ F(t_{k-1}) - gH(q_{k-1}^{i} - q_{c}) \right] \Delta t, 1 \leq k \leq N, 1 \leq j \leq M,
$$
 (5)

where  $\Delta t$  denotes the time step,  $t_k = k\Delta t$ ,  $\Delta l$  the space step,  $l_i = i\Delta l$ , *k* the time level, *i* the space grid point,  $M+1 = (L/\Delta l) + 1$  the total number of space discrete points, and  $N = T / \Delta t$  the total time levels in integration. Besides, some values are prescribed as  $M = 20$ ,  $N = 100$ ,  $\Delta t = 0.01$ , and  $\Delta l = 0.05$ .

#### 2.2 *CNOP*

Let  $q_T$  and  $q_T + \tilde{q}_{T_N}$  be the solution to Eq.(2) with the initial value being  $q_0$  and  $q_0 + \tilde{q}_0$ respectively, that is, if the governing equation is defined as *M* , then  $q_T = M(q_0)$  $q_T + \tilde{q}_{T} = M (q_0 + \tilde{q}_0)$ . In this paper, the  $L^2$  norm is employed, that is,

$$
||q||^2 = \int_0^L q^2 dl \,, \tag{6}
$$

with the constraint  $\|\tilde{q}_0\| \le d$ , in which the initial perturbation  $\tilde{q}_0^*$  is called the conditional nonlinear optimal perturbation, if and only if

$$
\tilde{q}_0^* = \arg \max_{\|\tilde{q}_0\| \le d} J(\tilde{q}_0)
$$
\n(7)

$$
J(\tilde{q}_0) = \|M (q_0 + \tilde{q}_0) - M (q_0)\| = \|\tilde{q}_{\text{TN}}\|,
$$
 (8)

where  $\tilde{q}_{\scriptscriptstyle TN}$  presents the nonlinear evolution of the initial perturbation  $\tilde{q}_0$ .

Conventional optimization methods are based on gradient related information, such as BFGS, SPG2 and  $SQP$  <sup>[2-5]</sup>, and the gradient is provided by the conventional adjoint model.

## **3 GENETIC ALGORITHM**

The iteration procedure of using genetic algorithm as optimization algorithm to locate CNOP can be illustrated as follows.

**Step 0***.* Population initialization. Set generation  $i = 0$ , and initialize population  $P(0)$ , which is a set of solution guess,

$$
P(0) = \left[ Q_1^{(0)}, Q_2^{(0)}, \dots, Q_n^{(0)} \right]
$$
 (9)

$$
Q_k^0 = \left[ \tilde{q}_{0,k}^{(1)}, \tilde{q}_{0,k}^{(2)}, \cdots, \tilde{q}_{0,k}^{(m)} \right],
$$
 (10)

where  $Q_k^{(0)}$  denotes the *k* th individual of initial

generation, and  $\tilde{q}_0^{(j)}$  stands for the *j* th element of individual  $Q_k^{(0)}$ , which is the discrete value of initial perturbation  $\tilde{q}_0$ , *n* is the size of population, and *m* the number of discrete point of  $\tilde{q}_0$  in [0, L], that is,  $\tilde{q}_0^{(j)} = \tilde{q}_0(l_j), j = 1, 2, \dots, m, l_j = (j-1)\Delta l$ . The methods of generating the initial population are of several forms, such as stochastic generation, prior information, and prior information combined with stochastic generation, among which the principle of initialization is to increase the population diversity as greatly as possible.

**Step 1**. Evaluate the cost function  $J(Q_k^{(i)})$ , where  $k = 1, \dots, n$ , according to Eq.(7) by integrating model Eqs.(3)-(5) with initial condition  $q_0 + Q_k^{(i)}$ , and select the best individual  $Q_*^{(i)}$  of the current population. That is,

$$
Q_*^{(i)} = \arg\max_{1 \le k \le n} J\left(Q_k^{(i)}\right) \quad . \tag{11}
$$

**Step 2.** Check whether  $Q_*^{(i)}$  satisfies the stop criterion of

$$
\sum_{g=i-n}^{i-1} \left| J\left(Q_s^{(g+1)}\right) - J\left(Q_s^{(g)}\right) \right| < e \;, \tag{12}
$$

or  $i \ge i_m$ , where *n* and *e* are the given values, and  $i_m$  is the specified maximum generation. If it is satisfied, output  $Q_*^{(i)}$  and  $\tilde{q}_0^* = Q_*^{(i)}$ , and then stop the procedure; if not, go to Step 3.

**Step 3**. Select genetic operators. The operators are selected according to the selection probability  $p_s(Q_k^{(i)})$ , which determines whether the individual for number *k* is selected for the following operators. Here  $p_s(Q_k^{(i)})$  is defined as follows:

$$
p_{s}\left(Q_{k}^{(i)}\right) = \begin{cases} \frac{J\left(Q_{k}^{(i)}\right)}{\sum_{l=1}^{n} \left|J\left(Q_{l}^{(i)}\right)\right|}, d > kn \\ \frac{2(n-j)}{n(n+1)}, d \leq kn, \end{cases}
$$
(13)

where *d* stands for the number of individuals whose cost function is greater than the averaged one in the current generation, and  $k$  is a coefficient to balance proportion selection and ranking selection.

**Step 4**. Determine crossover operators. Suppose that the two individuals selected to mate are  $(i)$  $\mathcal{Q}_{\scriptscriptstyle k_{\rm l}}^{(i)}$ and  $Q_{k_2}^{(i)}$ where  $(i)$   $\Gamma$   $\approx$   $(1)$   $\approx$   $(2)$   $\approx$   $(m)$ 1  $10, k_1 \t 10, k_1 \t 10, k_1$  $\mathcal{Q}^{(i)}_{k_1} = \left[ \, \tilde{q}^{(1)}_{0,k_1}, \tilde{q}^{(2)}_{0,k_1}, \cdots , \tilde{q}^{(m)}_{0,k_1} \, \right] \;,\;\mathcal{Q}^{(i)}_{k_2} = \left[ \, \tilde{q}^{(1)}_{0,k_2}, \tilde{q}^{(2)}_{0,k_2}, \cdots , \tilde{q}^{(m)}_{0,k_2} \, \right]$ 2  $10, k_2 \t 10, k_2 \t 10, k_2$  $\mathbf{Q}_{k_2}^{(i)} = \left[ \, \tilde{q}_{0,k_2}^{(1)} \, , \tilde{q}_{0,k_2}^{(2)} \, , \cdots , \tilde{q}_{0,k_2}^{(m)} \, \right] \; ,$ and  $1 \le k_1, k_2 \le n$ . For *j*, two elements  $\hat{q}_{o,k_1}^{(j)}$  and  $\hat{q}_{o,k_2}^{(j)}$ are stochastically generated within

 $\left[ \hat{q}_0^{(j)} - a\Delta_j, \, \breve{q}_0^{(j)} + a\Delta_j \right]$ , where  $\hat{q}_0^{(j)} = \min \left\{ \tilde{q}_{0,k_1}^{(j)}, \tilde{q}_{0,k_2}^{(j)} \right\}$ ,  $\tilde{q}_0^{(j)} = \max \left\{ \tilde{q}_{0,k_1}^{(j)}, \tilde{q}_{0,k_2}^{(j)} \right\}$  and  $\Delta_j = \tilde{q}_{0,k_2}^{(j)} - \tilde{q}_{0,k_1}^{(j)}$ , and then we obtain  $\hat{Q}^{(i)}_{k_1} = \left[ \hat{q}^{(1)}_{o,k_1}, \hat{q}^{(2)}_{o,k_1}, \cdots, \hat{q}^{(m)}_{o,k_1} \right]$  $\hat{\mathbf{\mathcal{Q}}}_{k_{1}}^{(i)} = \left[ \, \hat{q}_{o, k_{1}}^{(1)}, \hat{q}_{o, k_{1}}^{(2)}, \cdots, \hat{q}_{o, k_{1}}^{(m)} \, \right] \qquad \quad \ \ ,$  $(i)$   $\Gamma_2(1)$   $\Delta(2)$   $\Delta(m)$ 2  $10, n_2 \t 10, n_2 \t 10, n_2$  $\hat{\mathbf{Q}}_{k_2}^{(i)} = \left[ \, \hat{q}_{o, k_2}^{(1)}, \hat{q}_{o, k_2}^{(2)}, \cdots, \hat{q}_{o, k_2}^{(m)} \, \right].$ 

**Step 5***.* Determine mutation operators. Suppose that the individual selected to mutate is  $\hat{Q}_k^{(i)}$ , then the *j*<sup>th</sup> element can be modified in the following two ways:

$$
\overline{q}_{0,k}^{(j)} = \tilde{q}_{0,k}^{(j)} + \Delta \left( i, \overline{\tilde{q}_0^{(j)}} - \tilde{q}_{0,k}^{(j)} \right) , \qquad (14)
$$

and

$$
\overline{q}_{0,k}^{(j)} = \tilde{q}_{0,k}^{(j)} - \Delta \Big(i, \tilde{q}_{0,k}^{(j)} - \underline{\tilde{q}_0}^{(j)}\Big), \qquad (15)
$$

where  $\tilde{q}_0^{(j)}$  is the upper boundary of  $\tilde{q}_{0,k}^{(j)}$ ,  $1 \le k \le n$ , and  $1 \le j \le m$ , while  $\tilde{q}_0^{(j)}$  is the lower one,

$$
\Delta(t, y) = yr \left( 1 - \frac{t}{T} \right)^b, \tag{16}
$$

where *r* is a stochastic number in  $[0,1]$ , and *b* a parameter used to determine the degree of uncertainty, and then we obtain  $Q_k^{(i+1)} = \hat{Q}_k^{(i)}$ .

**Step 6***.* Execute elitist reserving strategies. Copy the best *w* individuals in  $i<sup>th</sup>$  generation directly to replace the last bad *w* individuals, which are ranked by their cost functions in  $i+1$ <sup>th</sup> generation. The elitist reserving strategy is the basis of genetic algorithm's convergence. Again, go to Step 1.





 $-10$ 



### **4 NUMERICAL RESULTS AND ANALYSES**

## 4.1 *Reasons for failure of conventional adjoint method*

In order to reveal the impact of "on-off" switches on numerical solutions and cost functions, and for the sake of intuitively displaying the change of  $J(\tilde{q}_0)$ according to  $\tilde{q}_0$ , the *M* −1 dimensional components of  $\tilde{q}_0$  are fixed to 0 while the other two components are varied. Then  $J(\tilde{q}_0)$  is varied in the two dimensional space so that the variation of  $J(\tilde{q}_0)$  with respect to  $\tilde{q}_0$  can be easily observed in three dimensional space. For the reason that the nonlinear interactions between different gridpoints are greatly influenced by their distances, two schemes (namely S1 and S2) are adopted to select the two components left, i.e., in S1, the  $5<sup>th</sup>$  and  $17<sup>th</sup>$  gridpoints are selected, while in S2, the  $5<sup>th</sup>$  and  $7<sup>th</sup>$  gridpoints are selected.

The behavior of numerical solution *q* is illustrated in Fig.1a, which shows that the zigzag oscillation is obvious and that the evolutional solution is discontinuous. If there is no "on-off" switch in the governing equation, the cost function is continuous (Fig.1b). If there is, however, the behavior of  $J(\tilde{q}_0)$  is undesirably poor (Fig.1c-d), and the degrees of

 $\overline{\mathbf{C}}$  for  $\sqrt{J(u_r)}$ 

poorness are very different between Fig.1c and Fig.1d. The closer the distance between two gridpoints, the poorer the behavior of  $J(\tilde{q}_0)$  is.

The discontinuity of  $J(\tilde{q}_0)$  results in the nonexistence of its gradient  $\nabla_{\tilde{q}_0} J(\tilde{q}_0)$ , and therefore the adjoint model could not provide correct gradient for the optimization process. Hence it is impossible to update the current solution, which prevents the iteration process from converging to the optimal solution, eventually resulting in the failure of capturing CNOP. The related problems have been investigated in detail in the literature on variational data assimilation with parameterization "on-off" switches in governing equations (e.g., Xu<sup>[12,13]</sup>, Zou<sup>[14]</sup>, Mu<sup>[15,16]</sup>, and Zheng  $[17]$ ). So this paper aims at seeking a new optimization method and checking the effectiveness of GA in capturing CNOP with "on-off" switches.

### 4.2 *GA used to capture CNOP*

In order to check whether GA can capture CNOP, two schemes (S1 and S2) described in Section 4.1 are used to check the performance of GA in capturing CNOP with different degrees of poor behavior of  $J(\tilde{q}_0)$ . In Exp.1, the  $5<sup>th</sup>$  and  $17<sup>th</sup>$  gridpoints are selected, and in Exp.2, the  $5<sup>th</sup>$  and  $7<sup>th</sup>$  gridpoints are selected. In this study the penalty function is used to deal with the constraint  $\|\tilde{q}_0\| \le d$ , i.e.,

$$
\hat{J}(\tilde{q}_0) = J(\tilde{q}_0) - mP(||\tilde{q}_0|| - d)(||\tilde{q}_0|| - d),
$$
  
\n
$$
P(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases}
$$
 (17)

where *m* is an amplified coefficient, and  $\hat{J}(\tilde{q}_0)$  is used in the calculation of cost function in GA.

4.2.1 Exp.1

In this experiment,  $\tilde{q}_0^{(i)} = 0$  , where  $i = 0, \dots, 3, 5, \dots, 15, 17, \dots, M$ , and the initial condition of model (2) is  $(i)$   $(a)$  $\hat{q}_{_0}^{(i)}=q_{0}^{(i)}, i=0,\cdots,3,5,\cdots,15,17,\cdots,M~~,~~\hat{q}_{_0}^{(i)}=q_{0}^{(i)}+\tilde{q}_{0}^{(i)}$  $\hat{q}_{_0}^{(i)}=q_{0}^{(i)}+\tilde{q}_{0}^{(i)}$  , and  $i = 4, 16$ , where  $q_0^{(i)} = q_0(l_i)$ ,  $q_0(l_i) = 0.28 - 0.15 \sin(pi\Delta l/2)$ ,  $i = 0, \dots, M$ , and  $d = 0.02$ .

When the  $5<sup>th</sup>$  and  $17<sup>th</sup>$  grid points are selected, the conditional nonlinear optimal perturbation  $\tilde{q}_0^*$  is shown in Fig.2a, and the optimization trace of GA is illustrated in Fig.2b. In order to amplify the performance of GA in capturing CNOP, the initial population is arbitrarily configured far away from CNOP. It is shown in Fig.2b that during the optimization process, the population evolves, the best

individual  $Q_*^{(i)}$  of the current generation gradually tends to approach the CNOP, and after approximately forty generations  $Q_*^{(i)}$  converges to the CNOP. Meanwhile, the iteration solution of the conventional adjoint method stagnates within a domain not far away from the initial population. This shows the better performance of GA in capturing CNOP when 'on-off' switches are applied.



Fig.2 Conditional nonlinear optimal perturbation (CNOP) and the optimization trace of genetic algorithm (GA), where the shaded contour represents  $\log_{10} J(\tilde{q}_0)$ . To display clearly, the values less than  $10^{4.4}$  are replaced by the white color, the red line denotes the optimization trace of  $GA$ ,  $d = 0.02$ , *x*-axis and *y*-axis are amplified by  $10^3$  times, and both  $5<sup>th</sup>$  and  $17<sup>th</sup>$  grid points are selected. (a)  $\log_{10} J(\tilde{q}_0)$  and CNOP; (b)  $\log_{10} J(\tilde{q}_0)$  and the trace of GA.

#### 4.2.2 Exp.2

In this experiment,  $\tilde{q}_0^{(i)} = 0, i = 0, \dots, 3, 5, 7, \dots, M$ , and the initial condition of model equations  $(3-5)$  is  $(i)$   $(i)$  $\hat{q}_{_{0}}^{(i)}=q_{0}^{(i)},i=0,\cdots,3,5,7,\cdots,M\,\,,\hat{q}_{_{0}}^{(i)}=q_{0}^{(i)}+\tilde{q}_{0}^{(i)}$  $\hat{q}_0^{(i)} = q_0^{(i)} + \tilde{q}_0^{(i)}, i = 4, 6$ .

Fig.3a-3b illustrate that although the behavior of cost function in Exp.2 is much poorer than that in Exp.1, GA can still capture CNOP, showing good performance of GA when the gradient of cost function does not exist.



Fig.3 Same as Fig.2, but for both  $5<sup>th</sup>$  and  $7<sup>th</sup>$  gridpoints.

#### 4.2.3 Exp.3

The dependence of GA's performance on the initial population is examined in this numerical experiment. GA starts from a set of initial guess values (namely initial population), iterates according to the principle of selecting the superior and eliminating the inferior, and converges to the global optimal solution with high probability. The principle of population's initialization is to increase the population's diversity as much as possible. To solving the CNOP, the initial population is generated stochastically, and because of the stochastic numbers used, there are significant differences between every two runs, which are compared in this section. From the results (Fig.4), it can be seen that the best individual in Fig.4a is quite different from that in Fig.4b, whereas both can converge to CNOP. However, the iteration numbers vary from Fig.4a to Fig.4b; the closer the initial best individual to CNOP, the fewer steps of iteration are needed. Although different initial populations used to solve CNOP can converge to CNOP, great difference exists in computational cost, and therefore investigating reasonable methods of generating initial population is one of the approaches to improve the convergence speed of GA.

### **5 CONCLUSIONS AND DISCUSSIONS**

When moist physical process parameterizations are included in the governing equation, "on-off" switches will be present, and hence the cost function constructed according to physical problems of interest is discontinuous and the gradient of cost function does not exist, preventing the conventional adjoint method from providing correct gradient for the optimization process. However, in this case, the optimization of CNOP

search can be treated as a nonsmooth process. This study intends to apply genetic algorithm (GA) to capture CNOP with "on-off" switches in the governing equation, and the effectiveness by this method is examined with two numerical experiments. The result shows that GA is capable of capturing the CNOP in the context of the idealized model used in this study.



Fig.4 Same as Fig.3, but for the trace of current best solution during the iteration process of solving CNOP with GA.

One of important potential advantages of GA used in capturing CNOP is that it does not require any complicated adjoint model, and therefore complicated physical process can be included in the prediction model if it is allowed by computational resources. The operational prediction model, which contains various physical processes, can be directly adopted in the capture of CNOP, which enables the sensitive area and physical variables detected in typhoon adaptive observation to correctly reflect the sensitivity of the model.

At the same time, it should be kept in mind that the computational time consumed by GA is much longer than that of the conventional adjoint method, which is also the difference between the stochastic searching algorithm and deterministic searching algorithm. Genetic algorithm, which starts from a set of solution, needs much more runs of model integration as compared with the conventional adjoint method, resulting in much more consumed time of computation when single CPU is used. However, it is encouraging that parallel computation can be easily carried out in genetic algorithm, for the operators of different individuals in one generation are relatively independent. In particular, the model integrations of different

individuals, which dominate the computational time, are completely independent among individuals and can thereby be conducted with ease in different CPUs. Thereby, all these suggest that we take full advantage of rapidly developing computational parallel technology.

However, while GA has the above advantages used in capturing CNOP, it should be noted that the realization of GA comes with some skills, such as the setting of various parameters and the selection and design of genetic operators. It seems that there is generally not a single set of genetic operators which universally suit for every optimization problem. It is a feasible approach adopted by most researchers to improve GA's performance by designing and improving genetic operators based on professional knowledge of specific problems. However, with an operational prediction model, awesomely complicated as it is, it is much more difficult to improve the performance of GA based on the knowledge of real models than that of idealized models. Meanwhile, this study just adopts a very simple ideal model with "on-off" switches to check the effectiveness of GA in solving CNOP with physical parameterizations in the governing equation. For the convenience of comparison with optimal solution, only two space gridpoints are used, and the dimension of calculation is very limited in number. However, if a real prediction model is used with much more calculation dimensions and complicated "on-off" processes, can GA still perform as well, or are there any intelligent optimization algorithms with much better performance and much lower computational cost? These problems can be dealt with only after a lot of investigations on performance comparison among different optimization algorithms as well as convergence precision and speed.

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