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ANALYSES OF ERRORS IN MEDIUM-TERM NUMERICAL FORECAST PRODUCTS FOR THE SUBTROPICAL HIGH 1998

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ABSTRACT: By statistic and dynamic analyses, we have come to the following conclusions: (1) The ECMWF medium-term numerical forecast can forecast medium-term activity of subtropical high, and the accuracy rate of forecast cannot have large improvement by translational corrections. (2) The important cause for the ECMWF medium-term numerical forecast to have errors in 1998 is that the astronomical tide is not included in the model. (3) Two indexes are found from which it can be judged that ECMWF medium-term numerical forecast will have errors if the astronomical tide is ignored in the model : ① When the 54.7° line under the moon of the nodical month astronomical singularities coincides with the trough-line of the subtropical jet flow from 50°E to 150°E on the 500 hPa level at 2000 L.T. of the same day, and is approximately vertical (α>60°) with the isotherm, then the day 0 – 2 days after the appearance of the nodical month astronomical singularities is defined as the initial day. Then in three successive days after the initial day, ECMWF medium-term numerical forecast of the northern latitude of the 588 line at 120 °E will have continuous errors as large as two latitudes (7/9). Otherwise, it won't have continuous errors (13/13). ② Otherwise, if the 54.7 ° line is in the range of a low pressure between two high pressures, then there is a dispersive error on the day of the nodical month astronomical singularities (5/7). There is not any error (6/6) otherwise.

Key words: ECMWF medium-term numerical forecast; subtropical high; error analysis; astronomical tide; analysis of dynamics

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1 INTRODUCTION

It is well known that the accuracy of the forecasting of western Pacific subtropical high contributes much to the success of forecasts of medium-term precipitation during the raining season of eastern China. For the past few years, NWP products are becoming indispensable foundations for operational forecasters making medium-term forecasts, with the rapid modernization of meteorological means and on-going improvement of the NWP products. Models capable of producing NWP products cannot yield results that entirely represent the “true atmosphere” but have varying errors as compared with observed analysis fields, because they are subject to insufficient recognition of the principle of atmospheric dynamics and limitation of computer speed and capacity, together with unavoidable errors in initial values, errors caused by the expression of continuous function by means of finite difference, coverage that the computer fails to describe being smaller than gridpoint intervals and the failure to represent features due to lack of global data. To use and improve NWP products, many of the researchers and meteorologists at home and abroad have been using various schemes to study the errors of NWP products for their causes, with few addressing astronomically. On the other hand, the astronomical tidal force field (shortened as ATF field) has been shown to have possible major

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effects on the general circulation, based on some statistical results concerning the ATF field and large-scale weather change, especially with the subtropical high[1 - 4], but questions like when and on what conditions the effects will take place remain unanswered. Theoretic analyses addressing how the ATF field affects the mechanism of atmospheric motion is, in some way, a theoretic attempt[5], but never put into practice, saying nothing of identification of the errors of NWP products. In view of the need for precipitation forecasting during the raining seasons in eastern China, the current work discusses how to identify the errors of medium-term NWP products from Europe and China (T_{106}) from the angle of ATF field, using relevant data about the northern latitude of the 588 contour of the subtropical high at 120°E.

2 GENERAL ANALYSIS OF ERRORS OF TWO MEDIUM-TERM NWP PRODUCTS

Following conventional verification techniques for judging “true” (T) and “false” (F) and according to error standards less than 2 and 5 latitudes, the T_{106} and ECMWF products as well as persistent extrapolations of the analysis fields of the T_{106} products are verified based on the northern latitude of the 588 contour determined from the real field of the T_{106} at 120°E. The results are presented in Tab.1 and Tab.2.

Tab.1 Evaluation of northern latitude of the 588 contour as predicted by T_{106} and ECMWF that is less than 2 latitudes in error at 120 º E (May 1 – September 30, 1998)

Valid period / h		72		96		120		144	
Results		T	F	T	F	T	F	T	F
T_{106}	f.	81	72	68	85	59	94	52	101
	%	52.9	47.1	44.4	55.6	38.6	61.4	34.0	66.1
ECMWF	f.	113	40	100	53	89	64	75	78
	%	73.9	26.1	65.4	34.6	58.2	41.8	49.0	51.0
Persistent extrapolation	f.	61	89	52	97	55	93	55	92
	%	40.7	59.3	34.9	65.1	37.4	62.8	37.4	62.6

Tab.2 Same as Table 1 but for errors less than 5 latitudes

Valid period / h		72		96		120		144	
Results		T	F	T	F	T	F	T	F
T_{106}	f.	120	33	105	48	100	53	94	59
	%	78.4	21.6	68.6	31.4	65.4	34.6	61.4	38.6
ECMWF	f.	142	11	141	12	136	17	127	26
	%	92.8	7.2	92.2	7.8	88.9	11.1	83.0	17.0
Persistent extrapolation	f.	101	49	99	50	92	56	87	60
	%	67.3	32.7	66.4	33.6	62.2	37.8	59.2	40.8

It is known from Tabs.1 & 2 that the ECMWF products are more capable of predicting the subtropical high activity than the T_{106} NWP products or the persistent extrapolations of the T_{106} analysis fields, no matter whether the standard is involved with 2 or 5 latitudes. Particularly in the 72-h and 96-h forecasts, the ECMWF is 73.9% and 65.4% correct in forecasting the northern boundary (of latitude) of the 588 contour at 120°E, as far as the 2-latitude error standard is concerned. The accurate rate is 92.8% and 92.2% respectively for the 5-latitude error standard. It justifies the relevance of relying, to a large extent, on the ECMWF for the forecasting of medium-term activity of the subtropical high. Then, the distribution is studied of cases with errors ≥ 2 latitudes in predicted northern boundary (Tab.3). It shows that samples forecasting northward and southward latitudes are almost the same in number, indicating that the ECMWF products yield mainly random errors so that its systematic errors are negligible. It is then not likely that corrections by horizontal northward or southward shift of the forecast values may have much improvement of the accuracy.

Tab.3 Distribution of larger-than-2-latitude errors of northern boundary at 120°E predicted by ECMWF

Valid period / h	72	96	120	144
Sample size	40	53	64	78
Northward forecast	15	28	32	41
Southward forecast	25	25	32	37

3 LARGE ERRORS CAUSED BY DROPPING ASTRONOMICAL TIDES

Analyzing the 40 samples in which the 72-h forecast of the northern boundary of the 588 contour at 120°E is equal to or larger than 2 latitudes (similar results are with the 53 samples for the 96-h forecast but are not discussed here due to text limitation), we find that they can be divided into two kinds, one having continuous errors, in which the error larger than 2 latitudes lasts continuously for more than three days, the other having dispersive errors, in which the error larger than 2 latitudes lasts for less than two days. Tab.4 gives the result of the 72-h forecasts covering May 25 – 31, 1998.

Tab.4 Comparisons of the northern boundary of the 588 contour at 120 °E between the 72-h forecast and observation for May 25 – 31, 1998

date	Forecast value	observation	Error	Type of error	Initial day	Singular points of nodical month
25	<20	17.5	0.0	True		
26	25.0	22.3	2.5	Continuous errors	Initial day	$\beta 0$
27	27.5	25.3	2.2	Continuous errors		
28	25.0	20.8	4.2	Continuous errors		
29	20.0	21.5	−1.5	True		
30	20.0	25.9	−5.9	Dispersive errors	Initial day	
31	26.7	27.5	−0.8	true		

It is known from Tab.4 that continuous errors occur on May 26 – 28 while dispersive errors on May 30. Following the categorization method introduced above, the 40 samples with false forecasts can be grouped into 7 continuous errors (21 samples) and 16 dispersive errors (19 samples). The samples are compared with corresponding astronomical data (Tab.5).

Tab.5 shows that the initial days of continuous errors all occur within 0 – 2 days after the singular point of the nodical month (7/7), which is much higher than the natural probability of 3/7. χ^2 is used in the verification, which finds the a = 0.005 confidence test is passed with χ^2 = 9.33.

For the dispersive errors, however, five of them occur on the singular points of the nodical month (5/16), much higher than the natural probability of 1/7 and the frequency for other phases of the nodical month. $\boldsymbol{X}^2$ is used in the verification, which finds the $a = 0.005$ confidence test is passed with $\boldsymbol{X}^2 = 3.251$. It may indicate that the lack of astronomical ebbs and rises in the formulation of numerical models accounts for most of the facts (7 × 3 + 5 = 26) among 40 errors that are larger than 2 latitudes.

Tab.5 Distribution of frequency of initial day for continuous and dispersive errors with the phase of nodical month

time	−3 day	−2 day	−1 day	Singular point	+1 day	+2 day	+3 day	+4 day
Continuous errors				2	3	2		
Dispersive errors	1	3	2	5	0	2	2	1

4 DYNAMICS ANALYSIS

Reference [5] reports that a generalized vertical moist potential vorticity equation is obtained based on equation sets for viscosity-free, non-uniform saturated moist and free atmosphere motion[6-8].

$$\mathrm{d}s/\mathrm{d}t = s(A+B+C+D+F+G) \tag{1}$$

$$A=\left(\frac{\partial q^*}{\partial p}\right)^{-1}\frac{\partial}{\partial p}\left(\frac{(p/p_0)^{-R/C_p}}{Cp}\frac{dQ}{dt}\right) \tag{2}$$

$$B= f^{-1}\left(\frac{\partial q^*}{\partial p}\right)^{-1}\left(-\frac{\partial\Phi}{\partial y}\frac{\partial q^*}{\partial x}+\frac{\partial\Phi}{\partial x}\frac{\partial q^*}{\partial y}\right)\frac{\partial}{\partial p}\left(\frac{q}{q^*}\right) \tag{3}$$

$$C=\left(f+z\right)^{-1}\left(\frac{\partial\Phi}{\partial x}\frac{\partial}{\partial y}\left(\frac{q}{q^*}\right)-\frac{\partial\Phi}{\partial y}\frac{\partial}{\partial x}\left(\frac{q}{q^*}\right)\right) \tag{4}$$

$$D= f^{-1}\left(\frac{\partial q^*}{\partial p}\right)^{-1}\left(\frac{\partial E_y}{\partial p}\frac{\partial q^*}{\partial x}-\frac{\partial E_x}{\partial p}\frac{\partial q^*}{\partial y}\right) \tag{5}$$

$$F= f^{-1}\left(\frac{\partial q^*}{\partial p}\right)^{-1}\left(\frac{\partial q^*}{\partial y}\frac{\partial}{\partial p}\left(-\frac{\mathrm{d}u}{\mathrm{d}t}\right)-\frac{\partial q^*}{\partial x}\frac{\partial}{\partial p}\left(-\frac{\mathrm{d}v}{\mathrm{d}t}\right)\right) \tag{6}$$

$$G=\left(f+z\right)^{-1}\left(\frac{\partial u}{\partial p}\frac{\partial w}{\partial y}-\frac{\partial v}{\partial p}\frac{\partial w}{\partial x}\right) \tag{7}$$

$$s=\frac{q}{q^*}\left(\frac{\partial q^*}{\partial p}\right)\left(f+z\right) \tag{8}$$

$$q = T\left(p/p_0\right)^{-R/Cp} \tag{9}$$

$$q^* = q \cdot \exp\left(\frac{L}{C_p}\frac{q_s}{T}\left(\frac{q}{q_s}\right)^a\right) \tag{10}$$

in which S is the moist potential vorticity, A is the contribution to changes in S by diabatic heating (with the exception of latent heat by condensation), through the alteration of vertical stratification, B is the condensation latent heat that relates to vertically inhomogeneity of hydrostatic unbalance, C is the condensation latent heat that relates to horizontally inhomogeneity of hydrostatic unbalance and D is the ATF field through the alteration of stratification, F is the feedback contribution by wind field developments to the changes in moist potential vorticity, G is the contribution of vortex tubes to moist potential vorticity, and other terms have the same meaning as in [5]. The equation set is rewritten in a dimensionless form of

$$e\frac{\partial s_1}{\partial t_1} + R\left(u_1\frac{\partial s_1}{\partial x_1} + v_1\frac{\partial s_1}{\partial y_1}\right) + \frac{\Omega}{fH_p}\left(w_1\frac{\partial s_1}{\partial p_1}\right) = s_1\left(f^{-1}(A+B+C+D+F+G)\right) \tag{11}$$

where $\boldsymbol{\Omega}$ and H_p are the characteristic quantities of vertical velocity and scale in the p-coordinates and the rest is all used in the conventional sense. Applying dimension analysis to them, we find that the ATF field may play an important role in the evolution of general circulation so that moist potential vorticity becomes non-conservative to excite geostrophic deviation and perturbation during circulation adjustment, the consequence of which being failure of ECMWF forecasts for areas concerned. As necessary conditions, they not only suffice that

$$O(e) = O[f^{-1}(A+B+C+D+F+G)] \tag{12}$$

$$O[f^{-1}(A+B+C+D+F+G)] > \max[O(R), O(\Omega/fH_p)] \tag{13}$$

but also satisfy that

$$O(D) \geq O(A+B+C+F+G) \tag{14}$$

From Eqs.(2) – (7), we know that Terms B, C, F and G, the magnitude of which depend on the inbalance of the atmospheric system itself, can become small enough to reach dynamic equilibrium with the sum of Terms A and D, due to its own adaptation and development processes, if there is not any stimulation from perturbations of Term A of the heating field and transformation of Term D of the ATF field. Dynamic equilibrium of D, which is quite weak, with other terms will not be reached, unless Term A is sufficiently small. In fact, prior to the generation of non-conservation of moist potential vorticity, i.e. when the time scale corresponds to spatial scale in its variation, or, when

$$O(e) = \max[O(R), O(\Omega/fH_p)] \tag{15}$$

it should be followed by

$$O[f^{-1}(A+B+C+D+F+G)] \leq O(e) = \max[O(R), O(\Omega/fH_p)] \tag{16}$$

The magnitude of D, on the other hand, depends on E_x and E_y, the ATF field, as well as on the ratio of the horizontal gradient to the vertical one for the generalized moist potential temperature q^*, i.e. moist potential instability of the atmosphere. Using the expression for gravitational tide potential, we have

$$O(D) = O\left(\frac{\partial q^*/\partial r}{\partial q^*/\partial p}\frac{2E_0\Psi(\cos\Theta)\sin a}{afrg}\right) \tag{17}$$

where $\partial q^*/\partial r$ is the differential in the direction of normal of q^*, E_0 is the characteristic quantity of the ATF field, Ψ(cos Θ) is mainly a spherical triangle function that is related to the distance form the lunar zenith Θ, α is the included angle between the ATF field and q^*, a is the radius of the earth. Substituting the order of magnitude for a and E_0 and presenting it in a Z-coordinates, we have

$$O(D) = O\left(\frac{\partial q^* / \partial r}{\partial q^* / \partial Z}\Psi(\cos\Theta)\sin a 10^{-7}\right) \tag{18}$$

From Eqs.(16) and (18), we know that before the generation of non-conservative moist potential vorticity, in areas which

$$O\left(\frac{\partial q^* / \partial r}{\partial q^* / \partial Z}\Psi(\cos\Theta)\sin a\right) > 10^3 \max[O(R), O(W / fH)] \tag{19}$$

there is

$$O(D) > O(A + B + C + D + F + G) \tag{20}$$

i.e.

$$O(D) = O(A + B + C + F + G) > O(A + B + C + D + F + G) \tag{21}$$

It shows that Term D is in the same order of magnitude and reaches dynamic equilibrium with the sum of other terms. On the occasion, however, the dynamic equilibrium will be broken if changes take place in the ATF field, unless there is diabatic heating perturbation in addition to latent heat by condensation, and the following

$$\begin{aligned} &O(f^{-1}D) = O[f^{-1}(A + B + C + F + G)] = \\ &= O[f^{-1}(A + B + C + D + F + G)] > \max[O(R), O(\Omega / fH_p)] \end{aligned} \tag{22}$$

will appear. In other words, it satisfies Eqs.(12) – (14), which triggers a process in which the moist potential vorticity is no longer conservative and eventually leads to the failure of numerical predictions which consider no ATF fields.

5 TWO INDEXES LEADING TO FAILURE OF ECMWF FORECASTS FOR NOT TAKING ASTRONOMICAL TIDES INTO ACCOUNT

From the dynamics study above, we know that the ATF field may be an important factor in the evolution of the general circulation so that non-conservative moist potential vorticity takes place as in [1], geostrophic deviation and perturbation are excited and the circulation is adjusted. For a given forecast, the most probable temporal and spatial conditions for the ECMWF forecast to fail, if the astronomical tides are not included, are the times when the ATF field takes a U-turn change (corresponding to an astronomical singular point) and the area where Eq.(19) is satisfied. From the expression, we know that the area meeting these conditions is most probably the place where the maximum horizontal component gets the largest in the ATF field (with Ψ(cos Θ) being the largest, generally thought to be near 54.7° at the projected point under the moon), and $\frac{\partial q^*}{\partial r}\Big/\frac{\partial q^*}{\partial z}$ is large enough (It is known from Eq.(10) for q^* that the interface between condensation and non-condensation and areas where stratification is moist-neutral are usually with an active trough and make themselves susceptible to the conditions) and the horizontal component of the field is generally normal to the contours of q^* (sin α=1). In view of it, when we draw the 54.7° line at the projected point under the moon (Line AB in Fig.1) for all of the 22 singular points of the nodical moon between May 1 and September 30, 1998, onto the 500-hPa weather maps at 2000 L.T. of the same days, two indexes can indeed be found that would foretell the failure of medium-term ECMWF forecasts. ① For the nine cases in which the 54.7° line coincides with the trough line of the subtropical high jet and forms an included angle (α>60°) with the isotherm (Fig.1), all of the seven continuous errors are taken in; for the 13 cases that do not meet the above conditions, however, none of the ECMWF predictions have continuous errors. ② For the 13 cases that do not meet the above conditions, five out of seven ECMWF forecasts have the dispersive errors on the same days of singular points of the nodical moon if the 54.7° line passes through the low pressure sector between subtropical highs (Fig.2). No errors occur in the other six cases that do not meet the two conditions.

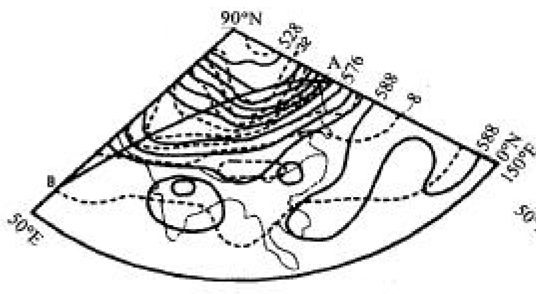

Fig.1 Synoptic chart at 500 hPa on May 20, 1998

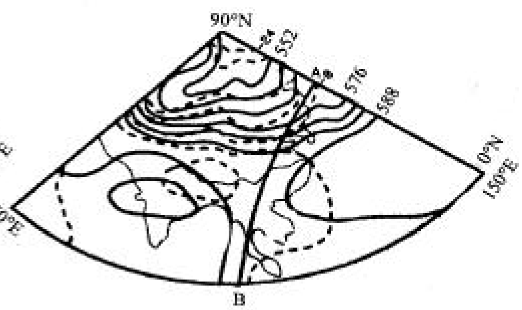

Fig.2 Same as Fig.1 but for June 29, 1998

Applying the two indexes to the raining seasons in 1999 and 2000, we find that among the 45 singular points of the nodical moon, three of the ECMWF medium-term forecasts have continuous errors of larger than 2 latitudes for consecutive 3 days in three singular points of the nodical moon that satisfy ①; none of 42 singular points that do not meet the condition of ① has continuous errors; fourteen out of eighteen singular points meeting the condition of ② have dispersive errors on the same days of the nodical moon while only one out of 24 singular points that do not meet the conditions of ① and ② has dispersive errors. In other words, the above two indexes are basically confirmed.

6 CONCLUSIONS AND DISCUSSIONS

a. The ECMWF medium-term numerical prediction products are an important basis for the forecasting of medium-term activity of the subtropical high but no significant improvement can be made to the accuracy by correcting the forecast values by way of horizontal shift.

b. Relatively large errors in the medium-term predictions by the ECMWF in 1998 may have been largely caused by negligence of the astronomical tides.

c. Two indexes are found in more intensive study that could indicate the failure of the medium-term forecasts due to the lack of astronomical tides in the model.

① When the 54.7° line under the moon of the nodical month astronomical singularities coincides with the trough-line of the subtropical jet flow from 50°E to 150°E on the 500 hPa level at 2000 L.T. of the same day, and has an included angle (α>60°) with the isotherm, then the day 0 – 2 days after the appearance of the nodical month astronomical singularities is defined as the initial day. Then in three successive days after the initial day, ECMWF medium-term numerical forecast of the northern latitude of the 588 line at 120 °E will have continuous errors as large as or more than two latitudes (7/9). Otherwise, it won't have continuous errors (13/13);

② With none of the above conditions are met, however, if the 54.7 ° line is in the range of a low pressure between two high pressures, then there is a dispersive error on the day of the nodical month astronomical singularities (5/7). There is not any error (6/6) otherwise.

d. The application in the raining seasons of 1999 and 2000 basically proves the two indexes.

Nonetheless, there are multiple reasons for relatively large errors in the ECMWF medium-term forecasts and the astronomical tide is only one of them. It is shown from the above dynamics analysis that the ATF field can have major effect on the general circulation only with certain circulation and specific astronomical conditions so as to cause large errors in the medium-term forecast models of the ECMWF, which do no include the ATF field. As a result, the results above do not mainly attribute the absence of the astronomical tides in the numerical models to the relatively large errors in the models. Take the raining season of 1998 for example. There are 26 errors which are identified by indexes ① and ② , taking 65% of the yearly total of 40; for the raining seasons of 1999 – 2000, however, there are only 23 such errors, accounting for 32% of the yearly total of 72.

It must be pointed out that statistical relationship and isolation technique for large errors, something like that between the astronomical tide and ECMWF numerical predictions, cannot be set up for the T_{106} numerical prediction products in China, because the affecting factors are more numerous and complicated to make the astronomical tide much less outstanding.

It must also be pointed out that the study above is only preliminary and lots of issues remain to be studied in greater depth. Why is there such good relationship between the nodical moon and large errors of the ECMWF medium-term forecasts while the link is less obvious between other periodicity of the tidal force and the latter? How do we make corrections to large errors that are likely to occur in the medium-term forecast of the ECMWF? What else causes them? All of these issues remain to be studied more thoroughly.

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