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DYNAMIC ANALYSIS OF INHERENT CAUSES FOR DEVIATION OF TROPICAL CYCLONE TRACKS

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ABSTRACT: To have a clearer picture of mechanisms responsible for the deviation of tropical cyclone (to be simplified as TC hereafter) tracks, the current work assumes the TC as a circular vortex with a radius of R. A general motion equation of TC is then determined by averaging its horizontal motion equation over the sentire region of TC. In the meantime, with the moving track of TC assumed as a characteristic arc, the curvature equation is derived for the track of movement and patterns of its deviation due to TC structure and variation are discussed. The result shows that the scale, size, maximum wind speed and radius are factors causing the deviation of TC tracks. In addition, asymmetric structure of TC is also important for the deviation of tracks. The results, achieved with hypothesis, agree with facts in some cases but disagree with them in others, which are to be verified with more observations or numerical simulations.

Key words: tropical cyclone; structure; deviation of tracks

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1 INTRODUCTION

The forecasting of movement has always been the core of TC research and routine operation. Generally speaking, the movement of TC is mainly dependent on the steering action of ambient field of airflow^[1]. In practice, the TC track tends to deviate and sometimes changes abruptly as it is subject to the steering current^[2]. As what some studies have pointed out, such deviation is related to the TC structure and its changes^[3]. Chen^[4] further suggests that the asymmetric structure of TC has significant impact on the routes by which it moves, especially when the ambient flow is weak, being one of the key factors leading to abnormal movement of the storm. Afterwards, other works^[5,6] conduct numerical experiments that investigate the relation between initial structures and tracks of movement and conclude that the movement of TC links with the vector of ventilation flow of the center. These attempts stop short of revealing the effects of ventilation flow on the TC movement. Up to the present, it has not been very clear concerning the TC structure and its changes in connection with track deviation. It is with this question that preliminary dynamic study is performed here in this work and some interesting understandings have been achieved.

2 BASIC EQUATIONS

The equations for description of TC horizontal motion are expressed as

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$$\begin{cases} \frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\partial f}{\partial x} + fv + F_u \\ \frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\partial f}{\partial y} - fu + F_v \end{cases}$$
(1)

where F_x and F_y are frictional force in the x and y direction, respectively and the rest are all meteorological symbols in the conventional sense.

On the assumption that TC is a circular vortex with a radius of R, the horizontal wind and pressure fields can be decomposed into as follows^[7]

$$\begin{cases} u = \overline{u} - v_q \sin q + u_r \cos q \\ v = \overline{v} + v_q \cos q + u_r \sin q \\ f = \overline{f} + f' \end{cases}$$
(2)

where v_q , u_r and f' are the tangential wind speed, radial wind speed and geopotential field of disturbance, respectively, and

$$\overline{F} = \frac{1}{\boldsymbol{p}R^2} \int_{0}^{2\boldsymbol{p}R} Fr \,\mathrm{d}r \,\mathrm{d}\boldsymbol{q}$$
(3)

is the mean of F over the entire region of TC.

Substituting Eq.(2) into Eq.(1) and seeking average over the TC region as in Eq.(3), we have $^{[7]}$

$$\begin{cases} \frac{\mathrm{d}\,\overline{u}}{\mathrm{d}\,t} = -\frac{\partial\overline{F}}{\partial x} + f_0\overline{v} + H_x + \overline{F}_u \\ \frac{\mathrm{d}\,\overline{v}}{\mathrm{d}\,t} = -\frac{\partial\overline{F}}{\partial y} - f_0\overline{u} - H_y + \overline{F}_v \end{cases}$$
(4)

where f_0 is the Corials parameter of the latitude where the center of TC locates, $(\overline{u}, \overline{v})$ can be taken as the moving speed of the TC center^[7,8], f is the ambient pressure field, and H_x and H_y are related with the structure of TC (horizontal wind field and its distribution), which are given by the following expression:

$$\begin{cases} H_x = \frac{\mathbf{b}}{\mathbf{p}R^2} \int_{0}^{2\mathbf{p}R} (v_q \cos \mathbf{q} + u_r \sin \mathbf{q})r^2 \sin \mathbf{q} \, \mathrm{d}r \, \mathrm{d}\mathbf{q} \\ H_y = \frac{\mathbf{b}}{\mathbf{p}R^2} \int_{0}^{2\mathbf{p}R} (-v_q \sin \mathbf{q} + u_r \cos \mathbf{q})r^2 \sin \mathbf{q} \, \mathrm{d}r \, \mathrm{d}\mathbf{q} \end{cases}$$
(5)

 $Xu^{[7]}$ et al. discuss in detail the effects of ambient pressure field and topographic friction on the tracks of TC. In this paper, the focus is on studying the role of TC structure and its latitudinal changes in deviating the track. It requires to change Eq.(4) into

$$\begin{cases} \frac{\mathrm{d}\,\overline{u}}{\mathrm{d}\,t} = f_0\,\overline{v} + H_x \\ \frac{\mathrm{d}\,\overline{v}}{\mathrm{d}\,t} = -f_0\,\overline{u} - H_y \end{cases}$$
(6)

Following a method similar to reference [7], we take the trajectory of TC as a characteristic arc, and its curvature (k) can be expressed using that of speed of TC movement:

$$k = \left(\overline{u}\frac{\mathrm{d}\overline{v}}{\mathrm{d}t} - \overline{v}\frac{\mathrm{d}\overline{u}}{\mathrm{d}t}\right) / \left(\overline{u}^2 + \overline{v}^2\right)^{\frac{3}{2}}$$
(7)

When k is increasing, it suggests that the TC track tends to have a cyclonic curvature, implying the possibility of deviating to the left (facing the moving direction of TC) in the Northern Hemisphere. When k is decreasing, it suggests that the TC track tends have an anti-cyclonic curvature, indicating the possibility of deviating to the right.

Substituting Eq.(6) into Eq.(7), we have

$$k = -\frac{f_0}{\sqrt{(\bar{u}^2 + \bar{v}^2)}} - \frac{\bar{u}H_y + \bar{v}H_x}{(\bar{u}^2 + \bar{v}^2)^{\frac{3}{2}}}$$
(8)

The first term on the right hand side of the above equation is the difference of TC track curvature across the latitude (i.e. with varying f_0). It means that increased latitude for a TC center is favorable for right curvature of the track (tending to have anti-cyclonic curvature), being consistent with the conclusion of reference [7]. Furthermore, as H_x and H_y are related with β (i.e. the latitude where the TC center locates), the effect of latitudinal variation on TC tracks should include the role of H_x and H_y . As shown in Eq.(8), with the increase of latitude, the curvature (deviation) of TC tracks is also related with the speed of the center and the structure. Discussions are held in this aspect.

It is not an unusual assumption to set the wind field of TC into symmetric and asymmetric components. Specifically, the symmetric component employs the Rankin vortex that has been modified^[9] and shown below and the asymmetric part is sinusoidal having wavenumber m.

$$v_{\boldsymbol{q}} = \begin{cases} \frac{v_m}{r_m} r[1 + a \sin m(\boldsymbol{q} + \boldsymbol{q}_0)], & 0 \le r \le r_m \\ v_m r_m \frac{1}{r} [1 + a \sin m(\boldsymbol{q} + \boldsymbol{q}_0)], & r_m \le r \le R \end{cases}$$

$$u_r = -bv_{\boldsymbol{q}} \tag{9}$$

where \mathbf{n}_m and \mathbf{g}_m are the maximum wind speed and radius respectively, *a* and *b* are constants larger than 0 but smaller than 1. *a* is the amplitude of the asymmetric component of the tangential wind speed of TC and *b* is associated with the magnitude of radial wind speed of TC.

Substituting Eq.(9) to Eq.(5), we have:

$$\begin{cases} H_x = -\frac{b\mathbf{b}_{m}r_m}{4R^2} (2R^2 - r_m^2) + \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin \mathbf{q} \cos \mathbf{q} \, \mathrm{d}\mathbf{q} - \frac{ab\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} \\ H_y = -\frac{\mathbf{b}_{m}r_m}{4R^2} (2R^2 - r_m^2) - \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} - \frac{ab\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} \\ H_y = -\frac{\mathbf{b}_{m}r_m}{4R^2} (2R^2 - r_m^2) - \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} - \frac{ab\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin \mathbf{q} \cos \mathbf{q} \, \mathrm{d}\mathbf{q} \\ H_y = -\frac{\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) - \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} - \frac{ab\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} \\ H_y = -\frac{\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) - \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} - \frac{ab\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} \\ H_y = -\frac{\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) - \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} - \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin m(\mathbf{q} + \mathbf{q}_0) \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q} \\ H_y = -\frac{\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) - \frac{a\mathbf{b}_{m}r_m}{4\mathbf{p}R^2} (2R^2 - r_m^2) \int_0^{2\mathbf{p}} \sin^2 \mathbf{q} \, \mathrm{d}\mathbf{q}$$

3 AXISYMMETRIC STRUCTURE

If TC is symmetric about the axis, which is equivalent to the wavenumber m = 0 in Eq.(9), the TC is then a modified Rankin vortex^[9]. Accordingly, Eq.(10) is reduced to:

$$\begin{cases} H_x = -\frac{bv_m r_m \mathbf{b}}{4R^2} (2R^2 - r_m^2) \\ H_y = -\frac{v_m r_m \mathbf{b}}{4R^2} (2R^2 - r_m^2) \end{cases}$$
(11)

Substituting Eq.(11) to Eq.(8) yields

$$k = -\frac{f_0}{\sqrt{\bar{u}^2 + \bar{v}^2}} + \frac{v_m r_m (2R^2 - r_m^2)(\bar{u} + b\bar{v})\mathbf{b}}{4R^2 \sqrt{(\bar{u}^2 + \bar{v}^2)^3}}$$
(12)

For convenience of discussions, individual factors on the right hand side of the equation above are analyzed in terms of their effects on deviation of TC tracks. In other words, singular factor and its influence on k is highlighted

3.1 Influence of latitude (Coriolis force and β effect) on deviation of TC track

From Eq.(12), we know that the influence of latitude on the deviation of tracks shows as the action of the Coriolis force and β effect. By differentiating the latitude at which the TC center locates, we obtain:

$$\boldsymbol{d}\boldsymbol{k} = -\frac{2\Omega}{\sqrt{\overline{u}^2 + \overline{v}^2}} \left[\cos \boldsymbol{j}_0 + \frac{v_m r_m (2R^2 - r_m^2)(\overline{u} + b\overline{v})}{4dR^2 (\overline{u}^2 + \overline{v}^2)} \sin \boldsymbol{j}_0 \right] \boldsymbol{d}\boldsymbol{j}$$
(13)

where Ω is the angular velocity of the earth rotation, *d* is the radius of the earth and \mathbf{j}_0 is the latitude where the TC center locates.

From Eq.(13), we know that for the Northern Hemisphere, the Coriolis force and β effect would make the north or northeast traveling TC to turn rightward ($d\mathbf{k} < 0$) with the increase of latitude ($d\mathbf{j} > 0$); the northwest going TC would also tend to turn right when latitude increases and $|\overline{u}| > b\overline{v}$, but it would turn left when latitude increases and $|\overline{u}| > b\overline{v}$ and $\mathbf{j}_0 < \mathbf{j}_k$. Specifically, \mathbf{j}_k is the critical latitude, which relates to the speed and direction of a moving TC as well as characteristic parameters such as the maximum wind speed, radius of gales and scale of TC:

$$\boldsymbol{j}_{k} = \mathrm{tg}^{-1} \frac{-4dR^{2}(\overline{u}^{2} + \overline{v}^{2})}{v_{m}r_{m}(2R^{2} - r_{m}^{2})(\overline{u} + b\overline{v})}$$
(14)

It is shown in the equation above that the decrease of TC scale (R), maximum wind speed (v_m)

and the radius of maximum winds (r_m) would increase \mathbf{j}_k provided that $r_m < (2/3)^{1/2} R$. During the northward movement of TC, the intensity usually decreases. If we take $a = 135^\circ$, R = 500 km, $v_m = 20$ m/s, $r_m = 100$ km, $\overline{u} = -10$ m/s, b = 0.1, then $\mathbf{j}_k \cong 89^\circ$. When $|\overline{u}| > b\overline{v}$, the northwest-traveling TC in the Northern Hemisphere always meets conditions of $\mathbf{j}_0 < \mathbf{j}_k$ to cause the track to deviate to the left.

It is then apparent that when the TC is moving at an angle of $\mathbf{a} \in (0, \mathbf{p} - \mathbf{d})$ with $\mathbf{a} = tg^{-1}(\overline{v}/\overline{u})$ and prescription that movement by due east is set at 0, due north at $\mathbf{p}/2$ and $tg\mathbf{d} = 1/b$, the TC tends to turn right with the increase of latitude; at an angle of $\mathbf{a} \in (\mathbf{p} - \mathbf{d}, \mathbf{p})$, a TC going northwestward would have the tendency to turn left with the increase of latitude. It may be one of the mechanisms by which TC migrating to middle-latitude regions mostly have parabola-shaped tracks.

3.2 Influence of angular velocity of earth rotation on deviation of TC track

On the basis of Eq.(12) and by seeking partial derivatives of Ω on both sides, we have:

$$dk = -\frac{2}{\sqrt{\bar{u}^{2} + \bar{v}^{2}}} \left[\sin j_{0} - \frac{v_{m} r_{m} (2R^{2} - r_{m}^{2})(\bar{u} + b\bar{v})}{4dR^{2} (\bar{u}^{2} + \bar{v}^{2})} \cos j_{0} \right] d\Omega$$
(15)

It is now obvious that the variation of the angular velocity of the earth rotation (Ω) also causes the deviation of TC tracks. As Ω varies at a relatively slow pace, it may be shown as variation of climatic background at inter-decadal or longer scales. Furthermore, the influence is also related with the latitude of TC, moving speed and direction as well as the scale, maximum wind speed and radius of maximum wind speed:

$$v_{m0} = \frac{4dR^2(\bar{u}^2 + \bar{v}^2) \operatorname{tg} \boldsymbol{j}_0}{r_m (2R^2 - r_m^2)(\bar{u} + b\bar{v})}$$
(16)

When the maximum wind speed of TC is smaller than v_{m0} that is determined by Eq.(16), any decrease of the angular velocity (Ω) will deviate the TC track to the left and otherwise is true.

3.3 Influence of variation of maximum wind speed on deviation of TC track

On the basis of Eq.(12) and by seeking differential of v_m on both sides, we have:

$$\boldsymbol{d}k = \frac{\Omega \cos \boldsymbol{j}_{0} r_{m} (2R^{2} - r_{m}^{2})(\overline{u} + b\overline{v})}{dR^{2} (\overline{u}^{2} + \overline{v}^{2})^{\frac{3}{2}}} \boldsymbol{d}v_{m}$$
(17)

It is obvious that with $\mathbf{a} \in [0, \mathbf{p} - \mathbf{d})$, a moving TC tends to turn right if the maximum wind speed decreases. It is consistent with the fact that TC, in the course of moving northward or northeastward, will weaken, the maximum wind speed will decrease and the track will mostly show the pattern of parabola curve after migrating into middle latitudes. For the TC with $\mathbf{a} \in (\mathbf{p} - \mathbf{d}, \mathbf{p})$, the decrease of maximum wind speed is favorable for left deviation of the track. The lower the latitude, the larger the scale of TC (R) will be. With $r_m \langle (2/3)^{(1/2)} R$ and smaller r_m , the deviation will be larger. 3.4 Influence of variation of radius of maximum wind speed on deviation of TC track

On the basis of Eq.(12) and by seeking differential of r_m on both sides, we have:

$$dk = \frac{\Omega \cos j_{0} v_{m} (\bar{u} + b\bar{v}) (2R^{2} - 3r_{m}^{2})}{2dR^{2} (\bar{u}^{2} + \bar{v}^{2})^{\frac{3}{2}}} dr_{m}$$
(18)

It is obvious that for a TC with $\mathbf{a} \in [0, \mathbf{p} - \mathbf{d})$, the track will be made easier to turn right if the radius of maximum wind speed $r_m \rangle (2/3)^{(1/2)} R$ and keeps increasing; the track is likely to turn left if $r_m \langle (2/3)^{(1/2)} R$ and r_m is increasing. On the other hand, opposite situations will occur when the TC is moving with $\mathbf{a} \in (\mathbf{p} - \mathbf{d}, \mathbf{p})$. Furthermore, the lower the latitude, the larger the maximum wind speed and TC scale, the more remarkable the deviation will be.

3.5 Influence of variation of radius of maximum wind speed on deviation of TC track

On the basis of Eq.(12) and by seeking differential of R on both sides, we have:

$$dk = \frac{\Omega \cos \mathbf{j}_{0} v_{m} r_{m}^{3} (\overline{u} + b\overline{v})}{d(\overline{u}^{2} + \overline{v}^{2})^{\frac{3}{2}}} dR$$
(19)

It is obvious that for a moving TC with $a \in [0, p - d]$, the track tends to turn right if the scale (R) gets small; it is likely to turn left if the scale grows during the movement. For a TC with $a \in (p - d, p)$, the track will turn left with the decrease of TC scale but turn right with the increase of TC scale.

3.6 Influence of variation of velocity of TC movement on deviation of TC track

On the basis of Eq.(12) and by seeking differential of \overline{u} on both sides, we have:

$$\boldsymbol{d}\boldsymbol{k} = \frac{\Omega}{2dR^2 \left(\overline{u}^2 + \overline{v}^2\right)^{\frac{5}{2}}} \left[4d\sin\boldsymbol{j}_0 R^2 \left(\overline{u}^2 + \overline{v}^2\right) \overline{u} + \cos\boldsymbol{j}_0 v_m r_m \left(2R^2 - r_m^2\right) \left(-2\overline{u}^2 - 3b\overline{u}\overline{v} + \overline{v}^2\right) \right] \boldsymbol{d}\overline{u} \quad (20)$$

It is obvious that an east-going TC may turn right when it moves faster ($d\overline{u} > 0$) and meets the condition of $\overline{u} < \operatorname{ctg} \mathbf{j}_0 v_m r_m \left(2R^2 - r_m^2\right) / \left(2dR^2\right)$; a northeast-going TC may turn left when its speed of northward movement keeps growing but beneath $\overline{v}(9b^2 + 8)^{(1/2)} / 4 - 3b\overline{v} / 4$; a northwest-going TC may turn left when it moves west at increasing speed (by absolute value) with $|\overline{u}| > 3b\overline{v} / 4 + \overline{v}(9b^2 + 8)^{(1/2)} / 4$; a west-going TC may turn left when the absolute value of moving speed increases.

Again, by seeking the differential of \overline{v} on both sides of Eq.(12), we have

$$\boldsymbol{d}\boldsymbol{k} = \frac{\Omega}{2dR^2 \left(\overline{u}^2 + \overline{v}^2\right)^{\frac{5}{2}}} \left[4d\sin\boldsymbol{j}_0 R^2 \left(\overline{u}^2 + \overline{v}^2\right) \overline{v} + \cos\boldsymbol{j}_0 v_m r_m \left(2R^2 - r_m^2\right) \left(b\overline{u}^2 - 3\overline{u}\overline{v} + 2b\overline{v}^2\right) \right] \boldsymbol{d}\overline{v} \quad (21)$$

It is obvious that a northeast-traveling TC may turn left if it moves north at increasing speed with $|\overline{v}| < \overline{u}(9+8b^2)^{(1/2)}/4b - 3\overline{u}/4b$; a north-traveling TC may turn right when it moves at increasing speed with $|\overline{v}| < \operatorname{ctg} \boldsymbol{j}_0 b v_m r_m (2R^2 - r_m^2)/(2dR^2)$; a northwest-traveling TC may

turn left when it moves north at increasing speed with $\overline{u}(9+8b^2)^{(1/2)}/4b-3\overline{u}/4b < \overline{v} < -\overline{u}(9+8b^2)^{(1/2)}/4b-3\overline{u}/4b$; it may turn right if its northward speed of movement is always within $\overline{v} < \overline{u}(9+8b^2)^{(1/2)}/4b-3\overline{u}/4b$ or $\overline{v} > -\overline{u}(9+8b^2)^{(1/2)}/4b-3\overline{u}/4b$.

4 ASYMMETRIC STRUCTURE

If TC has asymmetric structure, then the wavenumber $m \neq 0$ in Eq.(9).

4.1 Asymmetric distribution of dipole

As shown in some studies, the stream function field of TC is of significant dipole distribution. It can be inferred that the asymmetric component of the tangential and radial winds is of the same distribution. For this case, set the wavenumber of Eq.(9) at m = 1 and substitute it into Eq.(10), we have

$$\begin{cases} H_x = -\frac{bv_m r_m \mathbf{b}}{4R^2} (2R^2 - r_m^2) \\ H_y = -\frac{v_m r_m \mathbf{b}}{4R^2} (2R^2 - r_m^2) \end{cases}$$
(22)

It should be noted that the above equation is completely the same as Eq.(11). It shows that the asymmetric distribution of dipole in the TC wind field, i.e. the standard asymmetric wind field (or stream function field) with wavenumber 1 will not affect the movement of TC directly.

It is noted, too, that the dipole obtained with m = 1 in Eq.(9) is different from the so-called " β vortex pair" (The structure and observational evidence are presented in a separate paper). For instance, $v_q = u_r = 0$ at r = 0, i.e. there is no such thing being similar to the "ventilating air flow" between the " β vortex pairs" and thus no influence will be exerted on the track of TC.

4.2 Asymmetric distribution of multiple waves

Due to reasons like observations, asymmetric structure of TC has not been entirely revealed. Wilson^[8] reports in his study based on tower observation that there is multiple maximum wind speed within the troposphere over the TC. It was described that the eye of Hurricane Andrew (1992) was replaced by a larger eye wall as it moved across the Gulf of Mexico, as being scanned by the WSR-57 and WSR-88D weather radar in Miami. The eye wall then shrank and formed two mesoscale vortexes that were convectively active (the diameter being about 1 km at the altitude of 500 m), which were rotating around the center hurricane in an asymmetric structure of 2 waves^[9]. Similar structures are found during surveillance flights with the NOAA airplanes for Hurricanes Gladys (1975) and Hugo (1989)^[10]. It is therefore believed with sound basis that TC may be a system with asymmetric structure of multiple waves, i.e. m > 2. Then comes the question that consequently arises: Does the asymmetric structure have any influence on the TC track and how? By taking m = 2 (it is typical for TC to have a 2-wave asymmetric structure), the current work addresses this issue.

From Eq.(10), setting m = 2 will yield

$$\begin{cases} H_x = -\frac{v_m r_m (2R^2 - r_m^2) \mathbf{b}}{4R^2} (b - \frac{a}{2} \cos 2\mathbf{q}_0 + \frac{ab}{2} \sin 2\mathbf{q}_0) \\ H_y = -\frac{v_m r_m (2R^2 - r_m^2) \mathbf{b}}{4R^2} (1 + \frac{a}{2} \sin 2\mathbf{q}_0 + \frac{ab}{2} \cos 2\mathbf{q}_0) \end{cases}$$
(23)

Substituting the above equation to Eq.(8), we have

$$k = -\frac{f_0}{\sqrt{\overline{u}^2 + \overline{v}^2}} + \frac{v_m r_m (2R^2 - r_m^2) \mathbf{b}}{4R^2 \sqrt{(\overline{u}^2 + \overline{v}^2)^3}} \left[\overline{u} (1 + \frac{a}{2} \sin 2\mathbf{q}_0 + \frac{ab}{2} \cos 2\mathbf{q}_0) + \overline{v} (b - \frac{a}{2} \cos 2\mathbf{q}_0 + \frac{ab}{2} \sin 2\mathbf{q}_0) \right]$$
(24)

For the convenience of discussion, only the deviating effect by asymmetric TC structure on TC track will be studied here. Therefore, subtract Eq.(12) from Eq.(24) and denote the resulted difference as k_2 , then we have

$$k_{2} = \frac{av_{m}r_{m}(2R^{2} - r_{m}^{2})\boldsymbol{b}}{8R^{2}\sqrt{(\bar{u}^{2} + \bar{v}^{2})^{3}}} \left[\bar{u}(\sin 2\boldsymbol{q}_{0} + b\cos 2\boldsymbol{q}_{0}) + \bar{v}(-\cos 2\boldsymbol{q}_{0} + b\sin 2\boldsymbol{q}_{0})\right]$$
(25)

It is seen from Eq.(25) that the asymmetric structure can significantly cause changes in the curvature of TC tracks, i.e. influence its track of movement, when it is of 2-wave pattern. Furthermore, the influence (effect) is not only related with the latitude and scale of TC, maximum wind speed, radius of maximum wind speed and the moving speed of TC, but with the distribution (initial phase) of the 2-wave asymmetric structure as well as the amplitude.

In the Northern Hemisphere, a left deviation of track is favored if the initial phase of the 2-wave asymmetry satisfies that $(\overline{u} + b\overline{v})\sin 2\mathbf{q}_0 > (\overline{v} - b\overline{u})\cos 2\mathbf{q}_0$; on the contrary, a right deviation of track is favored if it meets the condition of $(\overline{u} + b\overline{v})\sin 2\mathbf{q}_0 < (\overline{v} - b\overline{u})\cos 2\mathbf{q}_0$.

In particular, a TC moving westward ($\overline{u} < 0$, $\overline{v} = 0$) may deviate to the right if the initial phase of the 2-wave asymmetric structure $2q_0 \in (2p - d, p - d)$ but to the left if $2q_0 \in (p - d, 2p - d)$. When the TC moves eastward ($\overline{u} > 0$, $\overline{v} = 0$), the opposite is true. For a northward-moving TC ($\overline{u} = 0$, $\overline{v} > 0$), the asymmetric structure, $2q_0 \in (p + d, d)$, will be a favorable condition for TC to deviate to the right, while the structure of $2q_0 \in (d, p + d)$ will be favorable for the left deviation of track.

5 CONCLUDSION AND DISCUSSIONS

Over the course of TC movement, changes in the latitude and scale of TC, maximum wind speed and radius of maximum wind speed can cause the TC track to deviate.

a. When a TC is moving northward with $a \in (0, p - d)$, in which movement towards due east is set 0 and due north is set p/2 when $a = tg^{-1}(\overline{v}/u)$ and $d = tg^{-1}(1/b)$, it tends to deviate to the right. When a TC is moving northwest with $a \in (p - d, p)$, it will deviate to the left with the increase of latitude.

b. When the maximum wind speed v_m is smaller than v_{m0} determined from Eq.(16), the decrease of Ω , the angular velocity of earth rotation, can lead to left deviation of the track and the increase of Ω can lead to deviation to the right.

c. For TC with $a \in [0, p - d]$, the movement will be deviated to the right if the maximum wind speed gets small; for TC with $a \in (p - d, p)$, the movement will be deviated to the left if the maximum wind speed gets small.

d. For TC with $\mathbf{a} \in [0, \mathbf{p} - \mathbf{d})$, the track tends to turn right if the radius of maximum wind speed $r_m > (2/3)^{(1/2)} R$ and keeps increasing; if the maximum wind speed $r_m < (2/3)^{(1/2)} R$, the track tends to turn left when r_m gets large. For TC with $\mathbf{a} \in (\mathbf{p} - \mathbf{d}, \mathbf{p})$, the scenario will be the opposite.

e. For TC with $a \in [0, p - d]$, the track will possibly be deviated to the right if the scale (*R*) gets small during the movement; For TC with $a \in (p - d, p)$, the track will possibly be deviated to the left if the scale (*R*) gets small. The opposite is true otherwise.

f. With the growth of eastward moving speed (its absolute value), TC moving east with $\overline{u} < \operatorname{ctg} \mathbf{j}_{0} v_{m} r_{m} (2R^{2} - r_{m}^{2})/(2dR^{2})$ will be deviated to the right; TC moving northeast with speed less than $\overline{v}(9b^{2} + 8)^{(1/2)}/4 - 3b\overline{v}/4$ will be deviated to the left; TC moving west or northwest with $|\overline{u}| > 3b\overline{v}/4 + \overline{v}(9b^{2} + 8)^{(1/2)}/4$ will be deviated to the left. With the growth of northward moving speed, TC moving northeast with $\overline{v} < \overline{u}(9 + 8b^{2})^{(1/2)}/4b - 3\overline{u}/4b$ will be deviated to the left and those with $\overline{v} < \operatorname{ctg} \mathbf{j}_{0} bv_{m} r_{m} (2R^{2} - r_{m}^{2})/(2dR^{2})$ will be deviated to the right; TC moving northwest will be deviated to the left if the speed satisfies $\overline{u}(9 + 8b^{2})^{(1/2)}/4b - 3\overline{u}/4b - 3\overline{u}/4b < \overline{v} < -\overline{u}(9 + 8b^{2})^{(1/2)}/4b - 3\overline{u}/4b$ but it will be deviated to the right if it satisfies $\overline{v} < \overline{u}(9 + 8b^{2})^{(1/2)}/4b - 3\overline{u}/4b$ or $\overline{v} > -\overline{u}(9 + 8b^{2})^{(1/2)}/4b - 3\overline{u}/4b$.

g. The asymmetric structure of TC multiple waves (wavenumber ≥ 2) can obviously deviate the track. For west-traveling TC ($\overline{u} < 0$, $\overline{v} = 0$), the track will be rightward deviated if the initial phase of the asymmetric structure $2\boldsymbol{q}_0 \in (2\boldsymbol{p} - \boldsymbol{d}, \boldsymbol{p} - \boldsymbol{d}]$ and it will be leftward deviated if $2\boldsymbol{q}_0 \in (\boldsymbol{p} - \boldsymbol{d}, 2\boldsymbol{p} - \boldsymbol{d}]$. When TC moves eastward ($\overline{u} > 0$, $\overline{v} = 0$), the scenario will be just the opposite. For north-going TC ($\overline{u} = 0$, $\overline{v} > 0$), the asymmetric structure of $2\boldsymbol{q}_0 \in (\boldsymbol{p} + \boldsymbol{d}, \boldsymbol{d})$ will be favorable for the track to turn right and that of $2\boldsymbol{q}_0 \in (\boldsymbol{d}, \boldsymbol{p} + \boldsymbol{d}]$ will be favorable for it to turn left.

h. When the 1-wave asymmetric structure of TC distributes as the dipole (taking m = 1) shown in Eq.(9), the track will not be deviated. It is attributed by the treatment in this work that the dipole is calm at the center of TC (the structure and observational evidence will be presented in separate work), i.e. the "ventilating air flow" is zero, having no impact on TC tracks.

It should be reminded that the analyzed results given in this work are obtained with a number of specific assumptions, such as distribution like the structure of Eq.(9), and some of them agree with existing observations. The following is a few citations. The track of northward or northeastward going TC is more likely to show the parabola pattern. As indicated by observations, TC usually weakens when it is northward located or migrates to areas of middle latitude by displaying as reduced scale, maximum wind speed and radius of maximum wind speed (and with $r_m < (2/3)^{(1/2)} R$). From the conclusions drawn from the dynamic study here, it is understood that the TC track meeting these conditions has the tendency of deviating to the right (Fig.1), which is why it shows a the parabola pattern. Other results are, however, to be verified with more observations or numerical simulations. As a matter of fact, it is still not fully clear what the asymmetric structure of TC is and how it affects the track and study is not common addressing the issue from the variation of latitude of TC center. These are all to be worked on further.



Fig.1 Schematic description of the effect of reduced scale, maximum wind speed and radius of tropical cyclone on its northward tracks

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