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A BAROTROPIC QUASI-GEOSTROPHIC MODEL WITH LARGE-SCALE TOPOGRAPHY, FRICTION AND HEATING

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ABSTRACT: Based on the barotropic equations including large-scale topography, friction and heat factor, a barotropic quasi-geostrophic model with large-scale topography, friction and heating is obtained by means of scale analysis and small parameter method. It is shown that this equation is a basic one, which is used to study the influence of the Tibetan Plateau on the large-scale flow in the atmosphere. If the friction and heating effect of large-scale topography are neglected, this model will degenerate to the general barotropic quasi-geostrophic one.

Key words: scale analysis; small parameter method; linearization; barotropic quasi-geostrophic model

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1 INTRODUCTION

The Tibetan Plateau occupies 1/4 of China mainland area with mean sea level elevation over 4000 m. It is the highest and most precipitous mountain in the world. Many researches (Ye and Gao, 1979; Qian, Yan and Wang et al., 1988; Wang, Qu and Yan et al., 1981; Qu, Wang and Qian et al., 1981; Li and Song, 1981; Wang, Wang and Qu et al., 1981; Zhang, Zhu and Zhu et al., 1988) show that the Tibetan Plateau influences the climate of China, East Asia, as well as the global atmospheric circulation. Recently many researches (Wu and Zheng, 1995) indicate that the effect of inter-monthly long term numerical forecast will be improved by including the topographic effect. Due to the topographic effect, plenty of large-scale cyclonic circulation and anti-cyclonic circulation occur in the lee of Tibetan Plateau and Taiwan Straits respectively. Topography plays an important role in the climate of China and the general circulation of the North Hemispheric atmosphere. So it is necessary to consider the dynamic effect, friction and heating effect of large-scale topography in the equations. Early in November, 1950, Charney and Neumann analyzed the numerical solution of the barotropic vorticity equation over a limited area of the earth's surface. Their results demonstrate that regarding the topographic effect in the barotropic equations is reasonable.

Due to the slope of large-scale topography, airflow is forced to rise and climb the ridge and this is the climbing effect. Sometimes airflow is forced to bypass the mountain, that is, the by-passing effect. Numerical experiments prove that (Wang, Qu and Yan et al., 1981; Qu, Wang and Qian et al., 1981), for both winter and summer, bypassing is the main dynamic effect of the Tibetan Plateau on zonal westerlies while climbing is the secondary effect.

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Qian and Yan et al. (1988) point out that the friction effect of topography includes horizontal friction and lateral friction. Horizontal friction can weaken surface layer system. Lateral friction is obvious in the north and south side of the Tibetan Plateau. It can weaken surface layer airflow, increase horizontal shear and change the distribution of the vorticity field. That many small-scale high pressure and low pressure systems respectively occur in North Plateau and South Plateau is due to lateral friction.

The heat effect of topography is also important. In the early 1950s,Ye, Luo and Zhu et al., (1957) and Zhu (1957) studied the structure of the streamline field around the Tibetan Plateau, the heat balance of the tropospheric atmosphere and the effect of large-scale topography by means of limited observational data. Both observational results and theoretical calculations prove that the Tibetan Plateau is a heat source in summer and a cold source in winter. The large-scale heat/cold source, which occurs in the middle troposphere, has influence on monsoon circulation, the break of 500 hPa summertime subtropical high pressure, as well as the formation and maintenance of 100 hPa South Asia high pressure. The heat effect of the Tibetan Plateau can be synchronous and lagging. It influences the plateau, East China and farther zones by means of teleconnection mechanism. The influence of heat effect on waves is also obvious. Taking topographic effects into account, a barotropic quasi-geostrophic atmospheric large-scale motion model with large-scale topography, friction and heating is established.

2 BASIC EQUATIONS

The barotropic model equations including topographic height h_s (x, y) can be given as

$$\begin{cases} \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv=-\frac{\partial f}{\partial x}-ru, \\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+fu=-\frac{\partial f}{\partial y}-rv, \\ (\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})(f-f_s)+(f-f_s)(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})=-Q, \end{cases} \tag{1}$$

where r is the Rayleigh friction coefficient, $f_s = gh_s$ (f_s is the topographic potential height), $f = gh$ (f is the free surface potential height), and Q is the heating ratio or mass transportation ratio. Eq.(1) is the barotropic model including topography, friction and heating.

Considering the atmospheric movement whose background is stationary, i.e. $u_0 = v_0 = 0$, then

$$\begin{cases} u=u',v=v', \\ f=f_0+f'=gH+f'=C_0{}^2+f', \end{cases} \tag{2}$$

with $C_0=\sqrt{gH}$ (H is the free surface height of stationary atmosphere), u',v',f' is the deviations of u,v,f from stationary state respectively. Thus the Eq.(1) can be rewritten as (Omitting "," for convenience):

$$\begin{cases} \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}-fv=-\dfrac{\partial f}{\partial x}-ru, \\ \dfrac{\partial v}{\partial t}+u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}+fu=-\dfrac{\partial f}{\partial y}-rv, \\ (\dfrac{\partial}{\partial t}+u\dfrac{\partial}{\partial x}+v\dfrac{\partial}{\partial y})(f-f_s)+(C_0{}^2+f-f_s)(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y})=-Q. \end{cases} \tag{3}$$

3 SCALE ANALYSIS

Considering the large-scale atmospheric movement（the horizontal scale $L\sim10^6 m$, the horizontal velocity scale $U\sim10m\cdot s^{-1}$, and the Coriolis parameter scale $f_0\sim10^{-4}s^{-1}$ ）, we set

$$\begin{cases} (x,y)=L(x_1,y_1), t=(L/U)t_1, (u,v)=U(u_1,v_1), \\ f=f_0ULf_1, f=f_0f_1, r=r_0r_1, Q=Q_0Q_1, f_s=f_0UL\alpha f_{s1}. \end{cases} \tag{4}$$

Specifically r_0 is the characteristic scale of Rayleigh friction coefficient $(r_0=10^{-1}f_0)$ and Q_0 is the characteristic scale of $Q(Q_0=f_0U^2)$. If the large-scale topography of the Tibetan Plateau is included, mean height is $3\sim4\times10^3$m approximately. Setting $f_s=f_0UL\alpha f_{s1}$, $(\alpha\sim1-10)$ and substituting Eq.(4) into Eq.(3), we get

$$\begin{cases} \dfrac{U^2}{L}(\dfrac{\partial u_1}{\partial t_1}+u_1\dfrac{\partial u_1}{\partial x_1}+v_1\dfrac{\partial u_1}{\partial y_1})-f_0U(f_1v_1)=-f_0U\dfrac{\partial f_1}{\partial x_1}-r_0U(r_1u_1), \\ \dfrac{U^2}{L}(\dfrac{\partial v_1}{\partial t_1}+u_1\dfrac{\partial v_1}{\partial x_1}+v_1\dfrac{\partial v_1}{\partial y_1})+f_0U(f_1u_1)=-f_0U\dfrac{\partial f_1}{\partial y_1}-r_0U(r_1v_1), \\ f_0U^2(\dfrac{\partial}{\partial t_1}+u_1\dfrac{\partial}{\partial x_1}+v_1\dfrac{\partial}{\partial y_1})(f_1-\alpha f_{s1}) \\ +[\dfrac{U}{L}C_0{}^2+f_0U^2(f_1-\alpha f_{s1})](\dfrac{\partial u_1}{\partial x_1}+\dfrac{\partial v_1}{\partial y_1})=-Q_0Q_1. \end{cases} \tag{5}$$

Dividing the first and second equations in Eq.(5) by f_0U, and the third equation in Eq.(5) by $\dfrac{C_0{}^2U}{L}$, we obtain

$$\begin{cases} R_0(\frac{\partial}{\partial t_1}+u_1\frac{\partial}{\partial x_1}+v_1\frac{\partial}{\partial y_1})u_1-f_1v_1=-\frac{\partial f_1}{\partial x_1}-\frac{r_0}{f_0}(r_1u_1), \\ R_0(\frac{\partial}{\partial t_1}+u_1\frac{\partial}{\partial x_1}+v_1\frac{\partial}{\partial y_1})v_1+f_1u_1=-\frac{\partial f_1}{\partial y_1}-\frac{r_0}{f_0}(r_1v_1), \\ R_0m_0{}^2(\frac{\partial}{\partial t_1}+u_1\frac{\partial}{\partial x_1}+v_1\frac{\partial}{\partial y_1})(f_1-af_{s1}) \\ +[1+R_0m_0{}^2(f_1-af_{s1})](\frac{\partial u_1}{\partial x_1}+\frac{\partial v_1}{\partial y_1})=-\frac{Q_0L}{C_0{}^2U}Q_1. \end{cases} \tag{6}$$

with

$$R_0=\frac{U}{f_0L}\text{，}\quad m_0{}^2=\frac{f_0{}^2L^2}{C_0{}^2}, \tag{7}$$

where R_0 and m_0^2 is the Rossby number and barotropic Obukhov number respectively.

4 SMALL PARAMETER METHOD

In the large-scale atmospheric movement , $R_0=10^{-1}$, we get

$$\begin{cases} u_1=u_1{}^{(0)}+R_0u_1{}^{(1)}+R_0{}^2u_1{}^{(2)}+\cdots, \\ v_1=v_1{}^{(0)}+R_0v_1{}^{(1)}+R_0{}^2v_1{}^{(2)}+\cdots, \\ f_1=f_1{}^{(0)}+R_0f_1{}^{(1)}+R_0{}^2f_1{}^{(2)}+\cdots, \end{cases} \tag{8}$$

where $u_1{}^{(0)},v_1{}^{(0)},f_1{}^{(0)}$ is the zero-order approximation of u_1,v_1,f_1 respectively， $u_1{}^{(1)},v_1{}^{(1)},f_1{}^{(1)}$ is the first-order approximation of u_1,v_1,f_1 respectively， $u_1{}^{(2)},v_1{}^{(2)},f_1{}^{(2)}$ is the second-order approximation of u_1,v_1,f_1 ⋯⋯. Noticing that

$$f=1+R_0b_1y_1 \quad (b_1=\frac{b_0L^2}{U}=1,\ b_0 \text{ is Rossby Parameter}) \tag{9}$$

Substituting Eq.(8) and Eq.(9) into Eq.(6) yields

$$
\begin{cases}
R_0 \dfrac{d}{dt_1}(u_1^{(0)} + R_0 u_1^{(1)} + \cdots) - (1 + R_0 b_1 y_1)(v_1^{(0)} + R_0 v_1^{(1)} + \cdots) \\
= -\dfrac{\partial}{\partial x_1}(f_1^{(0)} + R_0 f_1^{(1)} + \cdots) - \dfrac{r_0}{f_0} r_1 (u_1^{(0)} + R_0 u_1^{(1)} + \cdots), \\
R_0 \dfrac{d}{dt_1}(v_1^{(0)} + R_0 v_1^{(1)} + \cdots) + (1 + R_0 b_1 y_1)(u_1^{(0)} + R_0 u_1^{(1)} + \cdots) \\
= -\dfrac{\partial}{\partial y_1}(f_1^{(0)} + R_0 f_1^{(1)} + \cdots) - \dfrac{r_0}{f_0} r_1 (v_1^{(0)} + R_0 v_1^{(1)} + \cdots), \\
R_0 m_0^2 \dfrac{d}{dt_1}(f_1^{(0)} + R_0 f_1^{(1)} + \cdots - a f_{s1}) + [1 + R_0 m_0^2 (f_1^{(0)} + R f_1^{(1)} + \cdots - a f_{s1}) \\
[\dfrac{\partial}{\partial x_1}(u_1^{(0)} + R_0 u_1^{(1)} + \cdots) - \dfrac{\partial}{\partial y_1}(v_1^{(0)} + R_0 v_1^{(1)} + \cdots)] = -\dfrac{Q_0 L}{C_0^2 U} Q_1.
\end{cases}
\tag{10}
$$

Specifically

$$
\frac{d}{dt_1} = \frac{\partial}{\partial t_1} + (u_1^{(0)} + R_0 u_1^{(1)} + \cdots)\frac{\partial}{\partial x_1} + (v_1^{(0)} + R_0 v_1^{(1)} + \cdots)\frac{\partial}{\partial y_1}. \tag{11}
$$

From Eq.(10), the zero-order approximation equation can be derived:

$$
\begin{cases}
v_1^{(0)} = \dfrac{\partial f_1^{(0)}}{\partial x_1}, \\
u_1^{(0)} = -\dfrac{\partial f_1^{(0)}}{\partial y_1}, \\
\dfrac{\partial u_1^{(0)}}{\partial x_1} + \dfrac{\partial v_1^{(0)}}{\partial y_1} = 0.
\end{cases}
\tag{12}
$$

Eq.(12) can be rewritten as dimensional equations：

$$
\begin{cases}
f v^{(0)} = \dfrac{\partial f^{(0)}}{\partial x}, \\
f u^{(0)} = -\dfrac{\partial f^{(0)}}{\partial y}, \\
\dfrac{\partial u^{(0)}}{\partial x} + \dfrac{\partial v^{(0)}}{\partial y} = 0.
\end{cases}
\tag{13}
$$

Even the large-scale topography is included, the zero-order approximation equation reflects the geostrophic relation and horizontal non-divergence relation for large-scale atmospheric movement. If m_0^2, a is parameter, the first-order approximation of Eq.(10) is

$$\begin{cases} (\frac{\P}{\P t_1} + u_1^{(0)}\frac{\P}{\P x_1} + v^{(0)}\frac{\P}{\P y_1})u_1^{(0)} - b_1 y_1 v_1^{(0)} - v_1^{(1)} = -\frac{\P f_1^{(1)}}{\P x_1} - r_1 u_1^{(0)}, \\ (\frac{\P}{\P t_1} + u_1^{(0)}\frac{\P}{\P x_1} + v_1^{(0)}\frac{\P}{\P y_1})v_1^{(0)} + b_1 y_1 u_1^{(0)} + u_1^{(1)} = -\frac{\P f_1^{(1)}}{\P y_1} - r_1 u_1^{(0)}, \\ m_0^{\ 2}(\frac{\P}{\P t_1} + u_1^{(0)}\frac{\P}{\P x_1} + v_1^{(0)}\frac{\P}{\P y_1})(f_1^{(0)} - a f_{s1}) \\ + (1 - m_0^{\ 2} a f_{s1})(\frac{\P u_1^{(1)}}{\P x_1} + \frac{\P v_1^{(1)}}{\P y_1}) = -m_0^{\ 2} Q_1. \end{cases} \tag{14}$$

Restoring Eq.(14) to dimensional equation

$$\begin{cases} (\frac{\P}{\P t} + u^{(0)}\frac{\P}{\P x} + v^{(0)}\frac{\P}{\P y})u^{(0)} - b_0 y v^{(0)} - f_0 v^{(1)} = -\frac{\P f^{(1)}}{\P x} - r u^{(0)}, \\ (\frac{\P}{\P t} + u^{(0)}\frac{\P}{\P x} + v^{(0)}\frac{\P}{\P y})v^{(0)} + b_0 y u^{(0)} + f_0 u^{(1)} = -\frac{\P f^{(1)}}{\P y} - r v^{(0)}, \\ (\frac{\P}{\P t} + u^{(0)}\frac{\P}{\P x} + v^{(0)}\frac{\P}{\P y})(f^{(0)} - f_s) + (C_0^{\ 2} - f_s)(\frac{\P u^{(1)}}{\P x} + \frac{\P v^{(1)}}{\P y}) = -Q. \end{cases} \tag{15}$$

Combining the first and second equations of Eq.(15) into vorticity equation and rewriting the third equation of Eq.(15) would yield

$$\begin{cases} (\frac{\P}{\P t} + u^{(0)}\frac{\P}{\P x} + v^{(0)}\frac{\P}{\P y})(f + z^{(0)}) = -f_0 D - r z^{(0)}, \\ (\frac{\P}{\P t} + u^{(0)}\frac{\P}{\P x} + v^{(0)}\frac{\P}{\P y})(y - y_s) + \frac{C_0^{\ 2} - f_0 y_s}{f_0} D = -\frac{Q}{f_0}. \end{cases} \tag{16}$$

With

$$\begin{cases} u^{(0)} = -\frac{\P y}{\P y},\ v^{(0)} = \frac{\P y}{\P x},\ z^{(0)} = \frac{\P v^{(0)}}{\P x} - \frac{\P u^{(0)}}{\P y} = \nabla_h^{\ 2} j; \\ y = \frac{f^{(0)}}{f_0},\ y_s = \frac{f_s}{f_0},\ D = \frac{\P u^{(1)}}{\P x} + \frac{\P v^{(1)}}{\P y}. \end{cases} \tag{17}$$

Specifically y is the quasi-geostrophic stream function, y_s is the topographic stream function. From the second equation of Eq.(16), we get

$$D = \frac{f_0}{C_0^{\ 2} - f_0 y_s}[-(\frac{\P}{\P t} + u^{(0)}\frac{\P}{\P x} + v^{(0)}\frac{\P}{\P y})(y - y_s) - \frac{Q}{f_0}]. \tag{18}$$

Substituting Eq.(18) into the first equation of Eq.(16) yields

$$(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(f+\zeta^{(0)})=\frac{f_0^{\ 2}}{C_0^{\ 2}-f_0\psi_s}(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})(\psi-\psi_s)$$
$$+\frac{f_0}{C_0^{\ 2}-f_0\psi_s}Q-r\zeta^{(0)}. \tag{19}$$

Eq.(19) is the barotropic quasi-geostrophic equation with large-scale topography, friction and heating.

If topography, friction and heating effect is neglected, Eq.(19) can be reduced to

$$(\frac{\partial}{\partial t}+u^{(0)}\frac{\partial}{\partial x}+v^{(0)}\frac{\partial}{\partial y})q=0, \tag{20}$$

with

$$q=f+\zeta^{(0)}-\lambda_0^{\ 2}\psi=f+\nabla_h^{\ 2}\psi-\lambda_0^{\ 2}\psi. \tag{21}$$

Specifically q is the barotropic quasi-geostrophic potential vorticity. Eq.(20) is the law of barotropic quasi-geostrophic potential vorticity conservation.

5 LINEARIZATION

For the convenience of application, Eq.(19) is linearized. If the basic flow is regarded as the zonal mean flow, set

$$\begin{cases}\psi=\overline{\psi}+\psi',\ u^{(0)}=\overline{u}+u',v^{(0)}=v',\\ \zeta^{(0)}=\overline{\zeta}+\zeta',\ Q=\overline{Q}+Q',\end{cases} \tag{22}$$

where

$$\begin{cases}\overline{u}=-\frac{\partial\overline{\psi}}{\partial y},u'=-\frac{\partial\psi'}{\partial y},v'=\frac{\partial\psi'}{\partial x},\\ \overline{\zeta}=-\frac{\partial\overline{u}}{\partial y}=\frac{\partial^2\overline{\psi}}{\partial y^2},\zeta'=\frac{\partial v'}{\partial x}-\frac{\partial u'}{\partial y}=\nabla_h^{\ 2}\psi'.\end{cases} \tag{23}$$

where the variables with " ¯ " are zonal mean values that reflect the character of the large-scale atmospheric motion. " $\overline{u}$ " is the basic westerly flow.

Substituting Eq.(22) into Eq.(19) yields

$$[\frac{\partial}{\partial t}+(\overline{u}+u')\frac{\partial}{\partial x}+v'\frac{\partial}{\partial y}](f+\frac{\partial^2\overline{\psi}}{\partial y^2}+\nabla_h^{\ 2}\psi')$$
$$=\frac{f_0^{\ 2}}{C_0^{\ 2}-f_0\psi_s}[\frac{\partial}{\partial t}+(\overline{u}+u')\frac{\partial}{\partial x}+v'\frac{\partial}{\partial y}](\overline{\psi}+\psi'-\psi_s)$$
$$+\frac{f_0}{C_0^{\ 2}-f_0\psi_s}(\overline{Q}+Q')-r(\frac{\partial^2\overline{\psi}}{\partial y^2}+\nabla_h^{\ 2}\psi'). \tag{24}$$

Because the basic state conforms to Eq.(19), that is

$$(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x})(f+\frac{\partial^2\bar{\psi}}{\partial y^2})=\frac{f_0^{\ 2}}{C_0^{\ 2}-f_0\psi_s}(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x})(\bar{\psi}-\psi_s)+\frac{f_0}{C_0-f_0\psi_s}\bar{Q}-r\frac{\partial^2\bar{\psi}}{\partial y^2}. \tag{25}$$

then, by subtracting Eq.(25) from Eq.(24), we obtain

$$(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x})\nabla_h^{\ 2}\psi'+(\beta_0-\frac{\partial^2\bar{u}}{\partial y^2})\frac{\partial\psi'}{\partial x}=\frac{f_0^{\ 2}}{C_0^{\ 2}-f_0\psi_s}(\frac{\partial\psi'}{\partial t}+\frac{\partial\psi_s}{\partial x}\frac{\partial\psi'}{\partial y}-\frac{\partial\psi_s}{\partial y}\frac{\partial\psi'}{\partial x})+\frac{f_0}{C_0-f_0\psi_s}Q'-r\nabla_h^{\ 2}\psi'. \tag{26}$$

That is

$$(C_0^{\ 2}-f_0\psi_s)(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x})\nabla_h^{\ 2}\psi'+(C_0^{\ 2}-f_0\psi_s)(\beta_0-\frac{\partial^2\bar{u}}{\partial y^2})\frac{\partial\psi'}{\partial x}-f_0^{\ 2}(\frac{\partial\psi'}{\partial t}+\frac{\partial\psi_s}{\partial x}\frac{\partial\psi'}{\partial y}-\frac{\partial\psi_s}{\partial y}\frac{\partial\psi'}{\partial x})=f_0Q'-(C_0^{\ 2}-f_0\psi_s)r\nabla_h^{\ 2}\psi'. \tag{27}$$

Setting

$$\beta_1=\lambda_0^{\ 2}\frac{\partial\psi_s}{\partial y}, \qquad \beta_2=\lambda_0^{\ 2}\frac{\partial\psi_s}{\partial x}, \qquad \lambda_0^{\ 2}=\frac{f_0^{\ 2}}{C_0^{\ 2}}. \tag{28}$$

Specifically β_1 is the north-south topographic slope parameter, β_2 is the east-west topographic slope parameter, λ_0^{-1} is the barotropic Rossby deformation radius, then Eq.(27) can be reduced to

$$(C_0^{\ 2}-f_0\psi_s)(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x})\nabla_h^{\ 2}\psi'+(C_0^{\ 2}-f_0\psi_s)(\beta_0-\frac{\partial^2\bar{u}}{\partial y^2})\frac{\partial\psi'}{\partial x}-f_0^{\ 2}\frac{\partial\psi'}{\partial t}-C_0^{\ 2}(\beta_2\frac{\partial\psi'}{\partial y}-\beta_1\frac{\partial\psi'}{\partial x})=f_0Q'-(C_0^{\ 2}-f_0\psi_s)r\nabla_h^{\ 2}\psi'. \tag{29}$$

If topography, friction and heating are disregarded, then Eq.(29) degenerates into

$$(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x})\nabla_h^{\ 2}\psi'+(\beta_0-\frac{\partial^2\bar{u}}{\partial y^2})\frac{\partial\psi'}{\partial x}-\lambda_0^{\ 2}\frac{\partial\psi'}{\partial t}=0. \tag{30}$$

That is

$$(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x})q'+v'\frac{\partial\bar{q}}{\partial y}=0, \tag{31}$$

where

$$
\begin{cases}
\bar{q} = f + \dfrac{\partial^2 \bar{\psi}}{\partial y^2} - \lambda_0^2 \bar{\psi}, \\
q' = \nabla_h^2 \psi' - \lambda_0^2 \psi'.
\end{cases} \tag{32}
$$

Eq.(31）is the conservative law of quasi-geostrophic potential vorticity in the linearized barotropic model, where $\bar{q}, q'$ are the zonal mean potential vorticity and disturbance potential vorticity respectively. If the basic flow $\bar{u}$ =constant, Eq.(30) can be reduced to

$$
\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\nabla_h^2 \psi' + \beta_0 \frac{\partial \psi'}{\partial x} - \lambda_0^2 \frac{\partial \psi'}{\partial t} = 0. \tag{33}
$$

Eq.(29）is a linearized basic equation for analysis of the large-scale atmospheric motion with large-scale topography, friction and heating. In other papers, the influence of Eq.(29) on the large-scale atmospheric motion will be discussed.

6 CONCLUSIONS

a. The barotropic quasi-geostrophic model equations with the large-scale topography, friction and heating are Eq.(16) and Eq.(19). The linearized equation is Eq.(29).

b. If the large-scale topography, friction and heating effect is neglected, Eq.(19) will degenerate into a conservative form of the barotropic quasi-geostrophic potential vorticity Eq.(20), while Eq.(29) will degenerate into the linearized barotropic quasi-geostrophic model.

c. Eq.(19) and Eq.(29) are the basic equations in the discussion of large-scale atmospheric motion with large-scale topography, friction and heating effect. Omitting the function of β_2 (the east-west topographic slope parameter), Eq.(29) can be used to analyze the influence of the Tibetan Plateau on the large-scale atmospheric motion.

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