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THE SEMI-GEOSTROPHIC ADAPTATION PROCESS WITH TWO-LAYER BAROCLINIC MODEL IN LOW LATITUDE ATMOSPHERE

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ABSTRACT: In this paper, the adaptation process in low latitude atmosphere is discussed by means of a two-layer baroclinic model on the equator *b* plane, showing that the adaptation process in low latitude is mainly dominated by the internal inertial gravity waves. The initial ageostrophic energy is dispersed by the internal inertial gravity waves, and as a result, the geostrophic motion is obtained in zonal direction while the ageostrophic motion maintains in meridional direction, which can be called semi-geostrophic balance in barotropic model as well as semi-thermal-wind balance in baroclinic model. The vertical motion is determined both by the distribution of the initial vertical motion and that of the initial vertical motion tendency, but it is unrelated to the initial potential vorticity. Finally, the motion tends to be horizontal. The discussion of the physical mechanism of the semi-thermal-wind balance in low latitude atmosphere shows that the achievement of the semi-thermal-wind balance is due to the adjustment between the stream field and the temperature field through the horizontal convergence and divergence which is related to the vertical motion excited by the internal inertial gravity waves. The terminal adaptation state obtained shows that the adaptation direction between the mean temperature field and the shear flow field is determined by the ratio of the scale of the initial ageostrophic disturbance to the scale of one character scale related to the baroclinic Rossby radius of deformation. The shear stream field adapts to the mean temperature field when the ratio is greater than 1, and the mean temperature field adapts to the shear stream field when the ratio is smaller than 1.

Key words: adaptation process; semi-thermal-wind balance; vertical motion; internal inertial gravity waves

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1 INTRODUCTION

The idea of geostrophic adaptation process was first introduced by Rossby (1937; 1938). Later, with further study by Oboukhov (1949), Yeh (1957) and Zeng (1963) et al., the physical mechanism of geostrophic adaptation process in high-middle latitude atmosphere has been known clearly. In the study of barotropic model adaptation process in low latitude atmosphere, Chao (1996) and Lin and Chao (1997) presented the concept of semi-geostrophic adaptation, and pointed out that the ageostrophic atmospheric or oceanic disturbance in low latitude can reach zonal or meridional semi-geostrophic state through the semi-geostrophic adaptation process. Liu and Chao (1997) discussed the physical mechanism and the scale analysis of semi-geostrophic adaptation process in low latitude atmosphere using barotropic model. This paper will discuss the semi-geostrophic adaptation process in low latitude atmosphere using baroclinic model, especially the physical mechanism and the role of vertical motion in semi-geostrophic adaptation process.

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2 TROPICAL 2-LAYER BAROCLINIC MODEL AND EIGENFUNCTIONS

The linearized governing equations on the equator *b* plane can be written as

$$\begin{cases} \frac{\partial u}{\partial t} - byv + \frac{\partial f}{\partial x} = 0 \\ d\,\frac{\partial v}{\partial t} + byu + \frac{\partial f}{\partial y} = 0 \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \\ \frac{\partial}{\partial t}(\frac{\partial f}{\partial z}) + N^2 w = 0 \end{cases} \tag{1}$$

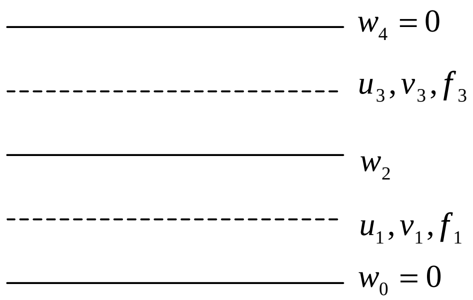

Fig.1 The schematic diagram of two-layer model

Where u, v and w are the velocity components in the eastward, northward and upward directions respectively. $\phi = p'/r_0$, p' is the pressure perturbation against static pressure, r_0 is density of static air. *N* is the brunt-väisälä frequency. ***d***=1 which is used to describe the semi-geostrophic adaptation process means zonal momentum equation does not satisfy the semi-geostrophic balance; while ***d***=0 which is used to describe the terminal adaptation state means zonal momentum equation satisfies the semi-geostrophic balance.

Using a two-layer model (Fig.1), we put dynamical equations and mass continuity equation on the first and the third levels, and put thermodynamic equation on the second layer, and substitute vertical differential by vertical difference. The vertical motion vanishes at the top and bottom.

$$\begin{cases} \frac{\partial u_1}{\partial t} - byv_1 + \frac{\partial f_1}{\partial x} = 0 \qquad \frac{\partial u_3}{\partial t} - byv_3 + \frac{\partial f_3}{\partial x} = 0 \\ d\,\frac{\partial v_1}{\partial t} + byu_1 + \frac{\partial f_1}{\partial y} = 0 \qquad d\,\frac{\partial v_3}{\partial t} + byu_3 + \frac{\partial f_3}{\partial y} = 0 \\ \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{w_2}{2\Delta z} = 0 \qquad \frac{\partial u_3}{\partial x} + \frac{\partial v_3}{\partial y} - \frac{w_2}{2\Delta z} = 0 \\ \frac{\partial}{\partial t}(\frac{f_3 - f_1}{2\Delta z}) + N^2 w_2 = 0 \end{cases} \tag{2}$$

It should be noted that the boundary conditions $w_0 = w_4 = 0$ have been used in continuity equation. By setting

$$u_s = \frac{1}{2}(u_3 - u_1) \qquad v_s = \frac{1}{2}(v_3 - v_1) \qquad f_s = \frac{1}{2}(f_3 - f_1) \tag{3}$$

then we can use (u_s, v_s) to describe the vertical shear stream field, i.e. baroclinic stream field; and use ϕ_s to describe the vertical shear pressure disturbance field corresponding with thickness field or mean temperature field. Thus, Eq.(2) can be rewritten as

$$\begin{cases} \frac{\partial u_s}{\partial t} - byv_s + \frac{\partial f_s}{\partial x} = 0 \\ d\,\frac{\partial v_s}{\partial t} + byu_s + \frac{\partial f_s}{\partial y} = 0 \\ \frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} - \frac{w_2}{2\Delta z} = 0 \\ \frac{\partial f_s}{\partial t} + \Delta z N^2 w_2 = 0 \end{cases} \tag{4}$$

Utilizing the last two equations of Eq.(4) to eliminate w_2, we can obtain

$$\begin{cases} \frac{\partial u_s}{\partial t} - byv_s + \frac{\partial f_s}{\partial x} = 0 \\ d\,\frac{\partial v_s}{\partial t} + byu_s + \frac{\partial f_s}{\partial y} = 0 \\ \frac{\partial f_s}{\partial t} + c_1^2\left(\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y}\right) = 0 \end{cases} \tag{5}$$

where

$$c_1^2 = 2(\Delta z)^2 N^2 \tag{6}$$

where c_1 is the characteristic velocity of internal inertial gravity waves. Therefore we get the equations which are formally the same as the barotropic model. By eliminating f_s, u_s from Eq.(5) we have

$$\mathsf{L}\, v_S = 0 \tag{7}$$

where

$$\mathsf{L} = \frac{\partial}{\partial t}\left[d\,\frac{\partial^2}{\partial t^2} - c_1^2\left(d\,\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + b^2 y^2\right] - bc_1^2\,\frac{\partial}{\partial x} \tag{8}$$

If we set

$$v_S = V_S(y)e^{i(kx - wt)} \tag{9}$$

where k is the wave number in x direction, w is angular frequency, Eq.(8) can be converted into

$$\frac{\mathrm{d}^2 V_S}{\mathrm{d}y^2} + \left[-\frac{bk}{w} + d\left(\frac{w^2}{c_1^2} - k^2\right) - \frac{b^2 y^2}{c_1^2}\right]V_S = 0 \tag{10}$$

In order to solve Eq.(10), we introduce the low latitude baroclinic Rossby radius

$$L_1 = \sqrt{\frac{c_1}{2b}} \tag{11}$$

and set

$$y = L_1 y_1 \qquad k = \frac{1}{L_1} k_1 \qquad w = \sqrt{2bc_1} w_1 = 2bL_1 w_1 \tag{12}$$

where y_1, k_1 and w_1 denote dimensionless quantities of y, k and w respectively. Eq.(10) is then converted into

$$\frac{\mathrm{d}^2 V_s}{\mathrm{d}y^2} + [-\frac{1}{2}\frac{k_1}{w_1} + d(w_1^2 - k_1^2) - \frac{1}{4} y_1^2] V_s = 0 \tag{13}$$

When Eq.(13) satisfies

$$V_s \,|_{y_1 \to \pm\infty} < \infty \tag{14}$$

its eigenvalues are

$$-\frac{1}{2}\frac{k_2}{w_2} + d(w_1^2 - k_1^2) = m + \frac{1}{2} \tag{15}$$

and the corresponding eigenfunctions are the Weber Functions

$$V_s(y) = D_m(y_1) = 2^{-\frac{m}{2}} e^{-\frac{1}{4}y_1^2} H_m(\frac{y_1}{\sqrt{2}}) = 2^{-\frac{m}{2}} e^{-\frac{b}{2c_1}y^2} H_m(\frac{y_1}{\sqrt{2}L_1}) \tag{16}$$

$$m = 0,1,2,\ldots$$

where H_m and D_m are Hermit polynomial and Weber function respectively.

The waves presented by Eq.(15) are mainly internal inertial gravity waves (contains mixed Rossby gravity waves or Yanai waves), baroclinic Rossby waves and Kelvin waves. Omitting w_1^2 from Eq.(15) yields that the angular frequency w_1 of low latitude baroclinic Rossby waves satisfies

$$w_1 = -\frac{k_1}{2dk_1^2 + 2m + 1} \qquad m = 0,1,2,\ldots \tag{17}$$

Its dimensional form is

$$w = -\frac{bk}{dk^2 + (2m+1)\frac{b}{c_1}} \qquad m = 0,1,2,\ldots \tag{18}$$

Taking $d = 0$, we can reduce Eqs.(17) and (18) to

$$w_1 = -\frac{k_1}{(2m+1)} \qquad m = 0,1,2,\ldots \tag{19}$$

$$w = -\frac{kc_1}{(2m+1)} \qquad m = 0,1,2,... \tag{20}$$

respectively, which are the dimensionless and the dimensional angular frequencies of baroclinic Rossby waves with the semi-geostrophic approximation (or long wave approximation). Furthermore, If taking $m = -1$, Eqs.(19) and (20) are respectively degenerated into

$$w_1 = k_1 \tag{21}$$

$$w = kc_1 \tag{22}$$

which are the dimensionless and the dimensional angular frequency of baroclinic Kelvin waves (requires v=0) in low latitude atmosphere.

Basing on the above analysis, we know that the evolution process in low latitude baroclinic atmosphere is mainly governed by baroclinic Kelvin waves and baroclinic long Rossby waves while the adaptation process is mainly dominated by internal inertial gravity waves, and the dispersion of inertia-internal gravity waves excited by initial semi-geostrophic disturbance leads to the establishment of the semi-geostrophic balance. Therefore, when we analyze the baroclinic adaptation process in low latitude atmosphere we can disregard the last term containing b on the right hand side of Eq.(8), namely neglect the differential of by with respect to y. Thus, the baroclinic Kelvin waves and baroclinic ultra long Rossby waves won't exist in semi-geostrophic adaptation process.

3 SOLUTION OF THE ADAPTATION EQUATION

Disregarding the change of by with y and using the third equation of Eq.(5), the first two equations of Eq.(5) can be converted into the following shear potential vorticity equation set

$$\frac{\partial q}{\partial t} = 0 \tag{23}$$

where

$$q(x,y) = (d\frac{\partial v_s}{\partial x} - \frac{\partial v_s}{\partial y}) - \frac{by}{c_1^2} f_s \tag{24}$$

is known as the baroclinic shear potential vorticity which is an invariant in time of semi-geostrophic adaptation in low latitude two-layer baroclinic model and depends completely on its initial value. Assuming that the initial values of u_s, v_s and f_s are u_{s0}, v_{s0} and f_{s0} respectively and owing to the initial state of adaptation process is ageostrophic, d=1, hence

$$q(x,y) = q_0(x,y) \equiv \frac{\partial v_{s0}}{\partial x} - \frac{\partial u_{s0}}{\partial y} - \frac{by}{c_1^2} f_{s0} \tag{25}$$

Utilizing Eq.(7) to solve the adaptation equation, and considering the adaptation equation is a fast development process, we neglect the last term on the right hand side of Eq.(7), namely, change L into $L^{(1)}$

$$L^{(1)} = \frac{\partial}{\partial t}[d\frac{\partial^2}{\partial t^2} - c_1^2(d\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) + b^2 y^2] \tag{26}$$

Integrating Eq.(7) with respect to time, we obtain

$$d\frac{\partial^2 v_s}{\partial t^2}-c_1^2(\frac{\partial^2 v_s}{\partial x^2}+\frac{\partial^2 v_s}{\partial y^2})+b^2y^2v_s=F(x,y) \tag{27}$$

where

$$F(x,y)=[\frac{\partial^2 v_s}{\partial t^2}-c_1^2(\frac{\partial^2 v_s}{\partial x^2}+\frac{\partial^2 v_s}{\partial y^2})+b^2y^2v_s]\Big|_{t=0} \tag{28}$$

Noting that when d=1, eliminating f_s by the second and the third equation of Eq.(5), and using the first one of Eq.(5), we get

$$\frac{\partial^2 v_s}{\partial t^2}-c_1^2\frac{\partial^2 v_s}{\partial y^2}=-by\frac{\partial u_s}{\partial t}+c_1^2\frac{\partial^2 u_s}{\partial x\partial y}=-b^2y^2v_s+by\frac{\partial f_s}{\partial x}+c_1^2\frac{\partial^2 u_s}{\partial x\partial y} \tag{29}$$

Therefore, Eq.(28) can be written as

$$F(x,y)=c_1^2\frac{\partial q_0}{\partial x} \tag{30}$$

If the initial values of v_s, $\frac{\partial v_s}{\partial t}$ are regarded as $F(x, y)$, $Y(x, y)$, then the definite problem of the semi-geostrophic adaptation process (so called the semi-thermal wind adaptation process) in low latitude baroclinic two-layer model can be written as

$$\begin{cases}\frac{\partial^2 v_s}{\partial t^2}-c_1^2(\frac{\partial^2 v_s}{\partial x^2}+\frac{\partial^2 v_s}{\partial y^2})+b^2y^2v_s=F(x,y)\\ v_s|_{t=0}=\Phi(x,y)\quad \frac{\partial v_s}{\partial t}|_{t=0}=\Psi(x,y)\end{cases} \tag{31}$$

For the corresponding eigenfunctions of the above equation are $D_m(y_1)=D(\frac{y}{L})$ if F=0, then we assume

$$(v_s,\Phi_s,\Psi_s,F_s)=\sum_{m=0}^{\infty}[v_{sm}(x,t),\Phi_{sm}(x),\Psi_{sm}(x),F_{sm}(x)]D(\frac{y}{L_1}) \tag{32}$$

Substituting Eq.(32) into Eq.(31), and noticing that

$$\frac{\mathrm{d}^2 D_m}{\mathrm{d}y_1^2}-\frac{1}{4}y_1^2D_m=-(m+\frac{1}{2})D_m \qquad y_1=\frac{y}{L_1} \tag{33}$$

or

$$\frac{\mathrm{d}^2 D_m}{\mathrm{d}y^2}-\frac{b^2y^2}{c_1^2}D_m=-\frac{b}{c_1}(2m+1)D_m \tag{34}$$

then the definite problem Eq.(31) can be rewritten as

$$\begin{cases}\frac{\partial^2 v_{sm}}{\partial t^2}-c_1^2\frac{\partial^2 v_{sm}}{\partial x^2}+(2m+1)bc_1v_{sm}=F_m(x)\\ v_{sm}|_{t=0}=\Phi_m(x)\quad \frac{\partial v_{sm}}{\partial t}|_{t=0}=\Psi_m(x)\end{cases} \tag{35}$$

Whose solutions are

$$v_{sm}(x,t)=\frac{1}{2}[\Phi_m(x+c_1t)+\Phi_m(x-c_1t)]$$
$$+\frac{1}{2c_1}\int_{x-c_1t}^{x+c_1t}\Psi_m(x)J_0(\frac{\sqrt{m+1/2}}{L_1}\sqrt{c_1^2t^2-(x-x)^2})dx$$
$$-\frac{\sqrt{(2m+1)bc_1}}{2}t\int_{x-c_1t}^{x+c_1t}\Phi_m(x)\frac{1}{\sqrt{c_1^2t^2-(x-x)^2}}J_1(\frac{\sqrt{m+1/2}}{L_1}\sqrt{c_1^2t^2-(x-x)^2})\mathrm{d}x$$
$$+\frac{1}{2c_1}\int_0^t\int_{x-c_1(t-t)}^{x+c_1(t-t)}F_m(x)J_0(\frac{\sqrt{m+1/2}}{L_1}\sqrt{c_1^2t^2-(x-x)^2})\mathrm{d}x\,dt \tag{36}$$

where J_0 and J_1 are the zero-order and the first-order Bessel Functions respectively. Obviously, If F_m, Y_m are not zero in finite domain, or If $|x|\to\infty$, F_m and $Y_m\to 0$, then when t is large enough Eq.(36) can be simplified into

$$v_{sm}(x,t)=\frac{1}{2c_1}\int_0^t\int_{-\infty}^{\infty}F_m(x)J_0\left(\frac{\sqrt{m+1/2}}{L_1}\sqrt{c_1^2t^2-(x-x)}\right)\mathrm{d}x\mathrm{d}t \tag{37}$$

Therefore, we have

$$\frac{\partial v_{sm}}{\partial t}=\frac{1}{2c_1}\int_{-\infty}^{\infty}F(x)J_0(\frac{\sqrt{m+1/2}}{L_1}\sqrt{c_1^2t^2-(x-x)^2})\mathrm{d}x \tag{38}$$

When t is large enough, it is clear that

$$\frac{\partial v_{sm}}{\partial t}\to 0 \qquad \frac{\partial v_s}{\partial t}\to 0 \tag{39}$$

which achieves the baroclinic semi-geostrophic balance, i.e. semi-thermal-wind balance.

4 SOLUTION OF VERTICAL MOTION AND PHYSICAL ANALYSIS OF SEMI-THERMAL-WIND BALANCE PROCESS

Eliminating u_s, v_s and f_s from Eq.(4), we have

$$\mathrm{L}^{(1)}w_2=0 \tag{40}$$

where

$$\mathrm{L}^{(1)}=d\frac{\partial^2}{\partial t^2}-c_1^2(d\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})+b^2y^2 \tag{41}$$

From Eq.(41) it is clear that in the two-layer baroclinic model the vertical motion is only governed by inertia-internal gravity waves, which was achieved through the horizontal convergence and divergence. If the initial values are $h(x,y)$ and $v(x,y)$, then the definite problem of the vertical velocity w_2 in low latitude baroclinic two-layer model is

$$\begin{cases} \frac{\partial^2 w_2}{\partial t^2} - c_1^2(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}) + b^2 y^2 w_2 = 0 \\ w_2|_{t=0} = h(x,y) \quad \frac{\partial w_2}{\partial t}|_{t=0} = v(x,y) \end{cases} \tag{42}$$

Expanding w_2, η, v in Weber function, we can obtain the solutions

$$\begin{cases} h = \sum_{m=0}^{\infty} h_m(x,t) D_m(\frac{y}{L_1}) \\ v = \sum_{m=0}^{\infty} v_m(x,t) D_m(\frac{y}{L_1}) \\ w_2 = \sum_{m=0}^{\infty} w_{2m}(x,t) D_m(\frac{y}{L_1}) \end{cases} \tag{43}$$

Therefore, we have

$$\begin{aligned} w_{2m}(x,t) &= \frac{1}{2}[h_m(x+c_1t) - h_m(x+c_1t)] \\ &+ \frac{1}{2c_1}\int_{x-c_1t}^{x+c_1t} v_m(x) J_0(\frac{\sqrt{m+1/2}}{L_1}\sqrt{c_1^2t^2-(x-x)^2})dx \\ &- \frac{\sqrt{(2m+1)bc_1}}{2} t \int_{x-c_1t}^{x+c_1t} h_m(x) \frac{1}{\sqrt{c_1^2t^2-(x-x)^2}} J_1(\frac{\sqrt{m+1/2}}{L_1}\sqrt{c_1^2t^2-(x-x)^2})dx \end{aligned} \tag{44}$$

It is obvious that in the two-layer model w_2 is only related to the initial vertical velocity and the initial vertical velocity tendency, but it is unrelated to the initial potential vorticity. Obviously, If η_m, v_m are not zero in finite field, or If $|x| \to \infty$, h_m and $v_m \to 0$, then when t is enough large we have $w_{2m} \to 0$, namely $w_2 \to 0$. Therefore, the atmosphere motion trends to be horizontal.

In view of the physical factors which can cause the vertical motion, differentiating the third equation of Eq.(4) with respect to time, and using the first two equations of Eq.(4) with d=1, we can obtain

$$\frac{\partial w_2}{\partial t} = 2\Delta z b y[(\frac{\partial v_s}{\partial x} - \frac{\partial u_s}{\partial y}) - \frac{1}{by}\nabla^2 f_s] = 2\Delta z b y(z_s - z_{gs}) \tag{45}$$

where

$$z_s = \frac{\partial v_s}{\partial x} - \frac{\partial u_s}{\partial y} \qquad z_{gs} = \frac{1}{by}\nabla_h^2 f_s \tag{46}$$

z_s is called the shear vorticity of flow field, z_{gs} is called the shear vorticity of temperature field. In the thermal-wind adaptation process, if the shear vorticity of temperature field is greater than that of flow field, then $\frac{\partial w_2}{\partial t} < 0$, the atmosphere follows the sinking motion; if the shear vorticity of the temperature field is smaller than that of the flow field, then $\frac{\partial w_2}{\partial t} > 0$, the atmosphere follows the lifting motion. In barotropic model, the semi-geostrophic balance is established through the lifting and the sinking of free surface caused by the surface inertial gravity waves, or through the convergence and divergence in the whole layer; while in baroclinic model, the establishment of semi-thermal balance is due to the adjustment between the stream field and the temperature field through the horizontal convergence and divergence which is related to the vertical motion excited by the internal inertial gravity waves. Therefore, there isn't essential difference of the role played by the vertical motion in the adaptation process between low latitude and middle-high latitude atmosphere.

5 TERMINAL ADAPTATION STATE

The terminal adaptation state must be semi-geostrophic balance with *d*=0, thus assuming that the terminal state of u_s, v_s and f_s are $u_{s\infty}$, $v_{s\infty}$ and $f_{s¥}$ respectively, and using Eqs.(4), (24) and (27), we have

$$\begin{cases} byu_{s\infty} = -\frac{\partial f_{s\infty}}{\partial y} \\ -\frac{\partial u_{s\infty}}{\partial y} - \frac{by}{c_1} f_{s\infty} = q_0 \\ -c_1^2 \frac{\partial^2 v_{s\infty}}{\partial y^2} + b^2 y^2 v_{s\infty} = F(x, y) \end{cases} \tag{47}$$

where the three formulas can be easily converted into

$$\begin{cases} \frac{\partial^2 u_{s\infty}}{\partial y^2} - \frac{b^2 y^2}{c_1^2} u_{s\infty} = -\frac{\partial q_0}{\partial y} \\ \frac{\partial^2 v_{s\infty}}{\partial y^2} - \frac{b^2 y^2}{c_1^2} v_{s\infty} = \frac{\partial q_0}{\partial x} \\ \frac{\partial^2 f_{s\infty}}{\partial y^2} - \frac{b^2 y^2}{c_1^2} f_{s\infty} = byq_0 \end{cases} \tag{48}$$

Assuming that the initial ageostrophic balance condition are

$$\begin{cases} u_s \big|_{t=0} = u_{s0}(x, y) \qquad v_s \big|_{t=0} = v_{s0} = 0 \\ f_s \big|_{t=0} = f_{s0}(x, y) \qquad byu_{s0} \neq -\frac{\partial f_{s0}}{\partial y} \end{cases} \tag{49}$$

Then from Eq.(24) we obtain

$$q_0(x,y) = -\frac{\partial u_{s0}}{\partial y} - \frac{by}{c_1^2} f_s \tag{50}$$

Setting

$$u'_{s0} = u_{s0} - u_{q0} = u_{s0} + \frac{1}{by}\frac{\partial f}{\partial y} \tag{51}$$

which is the initial value of geostrophic departure, and letting

$$\Delta u_s = u_{s\infty} - u_{s0} \qquad \Delta f_s = f_{s\infty} - f_{s0} \qquad \Delta v_s = v_{s\infty} - v_{s0} = v_{s\infty} \tag{52}$$

Thus, Eq.(48) can be converted into

$$\begin{cases} \dfrac{\partial^2 \Delta u_s}{\partial y^2} - \dfrac{b^2 y^2}{c_1^2}\Delta u_s = \dfrac{b^2 y^2}{c_1^2} u'_{s0} \\ \dfrac{\partial^2 \Delta v_s}{\partial y^2} - \dfrac{b^2 y^2}{c_1^2}\Delta v_s = -\dfrac{\partial}{\partial x}\left(\dfrac{\partial u_{s0}}{\partial x} + \dfrac{by}{c_1^2} f_0\right) \\ \dfrac{\partial^2 \Delta f_s}{\partial y^2} - \dfrac{b^2 y^2}{c_1^2}\Delta f_s = -by\dfrac{\partial u'_{s0}}{\partial y} \end{cases} \tag{53}$$

In order to elucidate the tendency of the adaptation process, we rewrite Eq.(48) as

$$\begin{cases} u_{s0} = U_{s0} e^{-\frac{x^2}{2L_x^2}} e^{-\frac{y^2}{2L_y^2}} \qquad v_{s0} = 0 \\ f_{s0} = \Phi_{s0} e^{-\frac{x^2}{2L_x^2}} e^{-\frac{y^2}{2L_y^2}} \end{cases} \tag{54}$$

This initial condition implies that: in the point $(x, y) = (0, 0)$, the maximum of u_{s0} and f_{s0} are U_{s0} and F_{s0} respectively; in the point $(x, y) = (\sqrt{2}L_x, \sqrt{2}L_y)$, the value of u_{s0} and f_{s0} are e^{-1} times of their directions respectively; and when $x \to \infty$ and $y \to \infty$, $u_{s0} \to \infty$ and $f_{s0} \to \infty$. Furthermore, the initial semi-ageostrophic balance needs $bu_{s0} \neq \frac{1}{L_y^2}\Phi_{s0}$. Substituting Eq.(54) into Eq.(53) we get

$$\begin{cases} \dfrac{\partial^2 \Delta u_s}{\partial y^2} - \dfrac{b^2 y^2}{c_1^2}\Delta u_s = \dfrac{b^2 y^2}{c_1^2}\left(U_{s0} - \dfrac{1}{bL_y^2}\Phi_{s0}\right) e^{-\frac{x^2}{2L_x^2}} e^{-\frac{y^2}{2L_y^2}} \\ \dfrac{\partial^2 \Delta v_s}{\partial y^2} - \dfrac{b^2 y^2}{c_1^2}\Delta v_s = -\dfrac{1}{L_x^2 L_y^2}\left(U_{s0} - \dfrac{bL_y^2}{c_1^2}\Phi_{s0}\right) xy e^{-\frac{x^2}{2L_x^2}} e^{-\frac{y^2}{2L_y^2}} \\ \dfrac{\partial^2 \Delta f_s}{\partial y^2} - \dfrac{b^2 y^2}{c_1^2}\Delta f_s = \dfrac{b}{L_y^2}\left(U_{s0} - \dfrac{1}{bL_y^2}\Phi_{s0}\right) y^2 e^{-\frac{x^2}{2L_x^2}} e^{-\frac{y^2}{2L_y^2}} \end{cases} \tag{55}$$

In order to solve Eq.(55), we expand Δu_s , Δv_s , Δf_s in Weber function, namely, set

$$((\Delta u_s),(\Delta v_s),(\Delta f_s)) = \sum_{m=0}^{\infty}[(\Delta u_s)_m, \frac{x}{L_x}(\Delta v_s)_m, (\Delta f_s)_m]e^{-\frac{x^2}{2L_x^2}}D_m(\frac{y}{L_1}) \tag{56}$$

Then substituting Eq.(56) into Eq.(55), and utilizing Eq.(34), we have

$$\begin{cases} \frac{b}{c_1}(U_{s0} - \frac{1}{bL_y^2}\Phi_{s0})y^2 e^{-\frac{y^2}{2L_y^2}} = \sum_{m=0}^{\infty}(2m+1)(\Delta u_s)_m D_m(\frac{y}{L_1}) \\ \frac{c_1}{bL_x L_y^2}(U_{s0} - \frac{bL_y^2}{c_1^2}\Phi_{s0})ye^{-\frac{y^2}{2L_y^2}} = \sum_{m=0}^{\infty}(2m+1)(\Delta v_s)_m D_m(\frac{y}{L_1}) \\ -\frac{c_1}{L_y^2}(U_{s0} - \frac{1}{bL_y^2}\Phi_{s0})y^2 e^{-\frac{y^2}{2L_y^2}} = \sum_{m=0}^{\infty}(2m+1)(\Delta f_s)_m D_m(\frac{y}{L_1}) \end{cases} \tag{57}$$

which are the expanding formulas in Weber function, applying the orthogonality of Weber function

$$\int_{-\infty}^{\infty} D_m(\frac{y}{L_1})D_n(\frac{y}{L_1})d(\frac{y}{L_1}) = \begin{cases} 0 & m \neq n \\ m!\sqrt{2p} & m = n \end{cases} \tag{58}$$

from Eq.(55) we can obtain

$$\begin{cases} (\Delta u_s)_m = \dfrac{-\frac{b}{c_1}(U_{s0} - \frac{1}{bL_y^2}\Phi_{s0})}{m!(2m+1)\sqrt{2p}}\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) \\ (\Delta v_s)_m = \dfrac{(U_{s0} - \frac{bL_y^2}{c_1^2}\Phi_{s0})}{m!(2m+1)\sqrt{2p}}\dfrac{c_1}{bL_x L_y^2} \\ (\Delta f_s)_m = \dfrac{c_1^2}{bL_y^2}(\Delta u_s)_m = \dfrac{c_1}{m^2}(\Delta u_s)_m = -\dfrac{\frac{c_1}{L_y^2}(U_{s0} - \frac{1}{bL_y^2}\Phi_{s0})}{m!(2m+1)\sqrt{2p}}\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) \end{cases} \tag{59}$$

Utilizing

$$D_n(x) = 2^{-\frac{n}{2}} e^{-\frac{x^2}{4}} H_n(\frac{x}{\sqrt{2}}) \tag{60}$$

and letting

$$m = \frac{L_y}{\sqrt{2}L_1} \tag{61}$$

we have

$$\begin{cases} \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) \\ \quad = \sqrt{2}\,2^{-\frac{m}{2}} L_1^2 (\frac{\sqrt{2m}}{\sqrt{1+m^2}})^2 \int_{-\infty}^{\infty} ye^{-y^2}\, 2(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y) H_m(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y)\mathrm{d}y \\ \int_{-\infty}^{\infty} ye^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})d(\frac{y}{L_1}) = 2^{-\frac{m}{2}+1} L_1 (\frac{\sqrt{2m}}{\sqrt{1+m^2}})^2 \int_{-\infty}^{\infty} ye^{-y^2} H_m(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y)\mathrm{d}y \end{cases} \tag{62}$$

Applying the following recursive relation formula of Hermite function in the first equation of Eq.(62)

$$2xH_m(x) = H_{m+1}(x) + 2mH_{m-1}(x) \tag{63}$$

we obtain

$$\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) = \sqrt{2}\,2^{-\frac{m}{2}} L_1^2 (\frac{\sqrt{2m}}{\sqrt{1+m^2}})^2 \int_{-\infty}^{\infty} ye^{-y^2} [H_{m+1}(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y) + 2mH_{m-1}(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y)]\mathrm{d}y \tag{64}$$

For the parity of function D_m is agree with the parity of m we can infer from Eq.(59) that

$$(\Delta u_s)_{2n+1} = 0 \qquad (\Delta v_s)_{2n} = 0 \qquad (\Delta f_s)_{2n+1} = 0 \tag{65}$$

Therefore, Eq.(62) can be rewritten as

$$\begin{cases} \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) \\ \quad = \frac{\sqrt{2}L_1^2}{2^n} (\frac{\sqrt{2m}}{\sqrt{1+m^2}})^2 \int_{-\infty}^{\infty} ye^{-y^2} [H_{2n+1}(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y) + 4nH_{2(n-1)+1}(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y)]\mathrm{d}y \\ \int_{-\infty}^{\infty} ye^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) = 2^{-\frac{2n+1}{2}+1} L_1 (\frac{\sqrt{2m}}{\sqrt{1+m^2}})^2 \int_{-\infty}^{\infty} ye^{-y^2} H_{2n+1}(\frac{\sqrt{2m}}{\sqrt{1+m^2}}y)\mathrm{d}y \end{cases} \tag{66}$$

By using

$$\int_{-\infty}^{\infty} ye^{-y^2} H_{2n+1}(ay)dy = \sqrt{p}\,\frac{(2n+1)!}{n!} a(a^2-1)^n \tag{67}$$

we can obtain the integral of Eq.(66)

$$
\begin{cases}
\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) \\
= \sqrt{2p}\, 2^{-n} L_1^2 (\frac{\sqrt{2}m}{\sqrt{1+m^2}})^3 [\frac{(2n+1)!}{n!}(\frac{m^2-1}{m^2+1})^n + \frac{(2n-1)!}{(n-1)!}(\frac{m^2-1}{m^2+1})^{n-1}] \\
\int_{-\infty}^{\infty} y e^{-\frac{y^2}{2L_y^2}} D_m(\frac{y}{L_1})\mathrm{d}(\frac{y}{L_1}) = \frac{2\sqrt{p}\, L_1(2n+1)!}{2^n n!} \frac{m^3}{(1+m^2)^{3/2}} (\frac{m^2-1}{m^2+1})^n
\end{cases}
\tag{68}
$$

Finally utilizing Eqs.(68), (58) and (67), we obtain the integrals of Eq.(57)

$$
\begin{cases}
(\Delta u_s)_{2n} = -\frac{(U_{s0} - \frac{1}{bL_y^2}\Phi_{s0})}{2^n(4n+1)} \frac{\sqrt{2}m^3}{(1+m^2)^{3/2}} [\frac{(2n+1)}{n!}(\frac{m^2-1}{m^2+1})^n + \frac{1}{2n(n-1)!}(\frac{m^2-1}{m^2+1})^{n-1}] \\
(\Delta v_s)_{2n+1} = \frac{(U_{s0} - \frac{bL_y^2}{c_1^2}\Phi_{s0})}{2^n n!(4n+3)} \frac{L_1}{L_x} \frac{2\sqrt{2}m}{(1+m^2)^3} (\frac{m^2-1}{m^2+1})^n \\
(\Delta f_s)_{2n} = -\frac{c_1(U_{s0} - \frac{1}{bL_y^2}\Phi_{s0})}{2^n(4n+1)} \frac{\sqrt{2}m}{(1+m^2)^{3/2}} [\frac{(2n+1)}{n!}(\frac{m^2-1}{m^2+1})^n + \frac{1}{2n(n-1)!}(\frac{m^2-1}{m^2+1})^{n-1}]
\end{cases}
\tag{69}
$$

which can be converted into

$$
\begin{cases}
(\Delta u_s)_{2n} = -\frac{(U_{s0} - \frac{1}{m^2}\frac{\Phi_{s0}}{c_1})}{2^n n!(4n+1)} \frac{\sqrt{2}m^3}{(1+m^2)^{3/2}} [(2n+1)(\frac{m^2-1}{m^2+1})^n + \frac{1}{2}(\frac{m^2-1}{m^2+1})^{n-1}] \\
(\Delta v_s)_{2n+1} = \frac{(U_{s0} - m^2\frac{\Phi_{s0}}{c_1^2})}{2^n n!(4n+3)} \frac{L_1}{L_x} \frac{2\sqrt{2}m}{(1+m^2)^3} (\frac{m^2-1}{m^2+1})^n \\
(\Delta f_s)_{2n} = -\frac{c_1(U_{s0} - \frac{1}{m^2}\frac{\Phi_{s0}}{c_1})}{2^n n!(4n+1)} \frac{\sqrt{2}m}{(1+m^2)^{3/2}} [(2n+1)(\frac{m^2-1}{m^2+1})^n + \frac{1}{2}(\frac{m^2-1}{m^2+1})^{n-1}]
\end{cases}
\tag{70}
$$

From Eq.(70), it is clear that $(\Delta u_s)_{2n}, (\Delta v_s)_{2n+1}$ and $(\Delta f_s)_{2n}$ tend to be zero quickly with the increasing of *n*, hence, it can reflect the variety of u_s, v_s and f_s between the initial adaptation state and the terminal adaptation state when a small *n* is taken.

The second formula of Eq.(70) shows that although the initial value $v_s = 0$, the terminal adaptation state of v_s isn't zero in the meridional shear field. From the second equation of Eq.(48), this is known to be caused by being nonzero of the initial baroclinic shear potential vorticity, which satisfies the governing condition of baroclinic shear potential vorticity conservation.

Comparing the first equation with the third one of Eq.(70), we can see clearly that when $L_y < \sqrt{2}L_1$, namely, m<0, the variation of $(\Delta u_s)_0$ is very small comparing with that of $(\Delta f_s)_0$, the mean temperature field adapts to the shear stream field; when $L_y > \sqrt{2}L_1$, namely, m > 0, the variation of $(\Delta f_s)_0$ is very small comparing with that of $(\Delta u_s)_0$, the shear stream field adapts to the mean temperature field.

6 CONCLUSIONS

From the analysis of the waves in low latitude baroclinic atmosphere, we can infer that the adaptation process in low latitude baroclinic atmosphere is mainly dominated by the internal inertial gravity waves. The initial semi-geostrophic disturbance is dispersed by inertia-internal gravity waves, which finally leads to the establishment of geostrophic balance in zonal direction, (the semi-geostrophic balance). In barotropic model, the adaptation process in low latitude atmosphere is mainly dominated by internal inertial gravity waves, the semi-geostrophic balance is achieved through the lifting and the sinking of free surface caused by the outer inertial gravity waves, or through the molar convergence and divergence in whole layer; while in baroclinic model, the establishment of semi-thermal balance is due to the adjustment between the stream field and the temperature field through the horizontal convergence and divergence which is related to the vertical motion excited by the internal inertial gravity waves. The vertical motion which is related to the internal inertial gravity waves is determined both by the distribution of the initial vertical motion and that of the initial vertical motion tendency, but it is unrelated to the initial potential vorticity. Finally, the semi-geostrophic adaptation makes the vertical motion horizontal.

The discussion of the terminal adaptation state shows that the adaptation direction between the mean temperature field and the shear flow field is determined by the ratio of the scale of the initial ageostrophic disturbance to the scale of one character scale related to the baroclinic Rossby radius of deformation. The shear stream field adapts to the mean temperature field when the ratio is greater than 1, and the mean temperature field adapts to the shear stream field when the ratio is smaller than 1.

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