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THE DIABATIC WAVES IN BAROTROPIC MODEL

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ABSTRACT: The equations of barotropic model are used to discuss the effects of diabatic factors such as heating of convective condensation, evaporation-wind feedback and CISK on the Rossby wave and the Kelvin wave. In low latitudes we have obtained the angular frequency and analyzed the period and stability of waves. The result shows the existence of the diabatic factors not only enlarges the period of adiabatic waves but also changes the stability of waves. Thus we think that the so-called intraseasonal oscillation and some other low-frequency oscillations are a kind of diabatic waves which are important factors producing the long-term weather changes and short-term climatic evolution.

Key words: low-frequency oscillation; diabatic wave; barotropic model

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1 INTRODUCTION

The 30-60 day low-frequency oscillations in the atmosphere have close relations with the long-term weather changes and the short-term climatic abnormality, so the low-frequency oscillations are important exploring field in atmospheric science research.

Early in the 1970s, Madden and Julian (1971) firstly found that there are 40-50 day low-frequency oscillations between the tropical atmospheric zonal wind field and surface pressure field by the spectrum analysis of ten years (1957-1967) in Canton Island. Then they (1972) confirmed not only the 40-50 day low-frequency oscillation which occur at all the tropical zones propagate eastward with the zonal wave-number being one but also the oscillations which have relations with the moving Walker circulation originate in from tropic Indian Ocean and Western pacific Ocean. Hereafter, Yasunari (1979,1980) pointed out that the cloud amount of Indian monsoon region has 30-40 day periodic variation on the base of the analysis of satellite cloud pictures. Krishnamurti (1982) analyzed MONEX' s data and verified that the monsoon through activities over South Asia have the 30-50 day oscillation with slow northward propagating. Murakami' s researches (1984,1985) revealed that the characters of eastward and northward propagating exist in the intraseasonal disturbance wind field and the pressure field. After a lot of low-frequency oscillations in the tropical atmosphere were found, meteorologists in succession discovered there are the 30-60 day in jet stream, polar vortex, subtropical high temperature and rainfall amount (and so on) in middle and high latitudes. As was pointed out by Krishnamurti (1985), the 30-60 day low-frequency oscillation is a kind of global atmospheric phenomenon.

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Biography: LI Li-ming (1976 -), male, native from Hefei City Anhui Province, graduate at Peking University, undertaking the study of dynamic meteorology.

In order to understand the rule and mechanism, meteorological researchers were widely engaged in dynamical researches of low-frequency oscillation. Analyzing the effects of the heating of condensation caused by the conditional instability on low latitudes atmospheric wave motion, Yamasaki (1969) and Hayashi (1970) pointed out that the wave motion can cause CISK mechanism as Ekman pumping. Then Lindzin (1974) concluded these researches with the wave-CISK theory and introduced it into the Ekman-CISK theory proposed by Charney (1964) and so on. Li (1983, 1985), Hayashi (1986), Lau (1987), Miyakara (1987), Chang (1988) and Liu (1990) further analyzed the effects of the wave-CISK on the low-frequency oscillation with theories and numerical models. In low latitudes zone, the massive oceanic bodies make evaporation and rainfall abundant, so on the base of interaction between SST, evaporation and atmosphere motions. Emanuel (1987) and Neelin (1987) simultaneously proposed the evaporation-wind feedback theory of atmospheric low-frequency oscillation. Then the evaporation-SST feedback theory was given by Lau (1988) and so on. Later, the joint effects of SST, evaporation-wind feedback (or wave-CISK) and evaporation-wind feedback on the low-frequency oscillation were explored in theories by Liu (1993), Li (1993), Zhao (1996) and so on.

As a whole, the dynamic mechanics of low-frequency oscillation can be concluded that the atmospheric response to the various heat sources (sea temperature, heating of condensation, wave-CISK, evaporation-wind feedback and so on). In essence, low-frequency oscillations are a kind of diabatic waves caused by diabatic factors. If the adiabatic waves, especially Rossby waves, are the important factors causing short-term weather changes (the period of Rossby wave is about a week), we think that diabatic waves, especially the diabatic Rossby wave and diabatic Kelvin waves, are the important factors causing the middle-term and long-term weather changes.

In this paper, the diabatic waves caused by diabatic factors such as the heating of convective condensation, evaporation-wind feedback and CISK are analyzed.

2 BASIC EQUATIONS

The equations of linear barotropic model including diabatic factors are

$$\begin{cases} \dfrac{\partial u}{\partial t} - fv = -\dfrac{\partial \Phi'}{\partial x} \\ d\,\dfrac{\partial v}{\partial t} + fu = -\dfrac{\partial \Phi'}{\partial y} \\ \dfrac{\partial \Phi'}{\partial t} + c_0^2\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) = -Q \end{cases} \tag{1}$$

where t is time, x and y represent east-west direction and south-north direction (the eastward and northward directions are positive), respectively, u and v represent zonal and meridional wind speeds, respectively, f is the Coriolis parameter, ϕ=gh represents gravitational potential of free surface $Z=h(x,y,t)$, and let

$$h = H + h' \tag{2}$$

where H represents the height of stationary free surface, h is the deviation to H, and so

$$c_0 = \sqrt{gh}, \qquad \Phi' = gh' \tag{3}$$

In Eq.(1),Q is the diabatic heating rate or mass transferring rate, δ is the filter parameter. For the wave-length of diabatic wave is very long in x direction, δ can be taken as 0 which the long-wave approximation. It can not only filter the high –frequency inertial gravitational wave but also keep the Rossby wave or Kelvin wave with long wave-length.

Three kinds of diabatic factors are discussed as follows:

2.1 *The heating of convective condensation*

If there is lots of vapor in the inferior atmospheric layer, the lifting caused by the inferior atmospheric horizontal convergence make vapor saturating then heating atmosphere. On this condition, we designate

$$Q_1 = -g\ c_0^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right), \qquad (0<g<1) \tag{4}$$

where g is a constant. In fact, there is so much seawater in oceans that the rise of SST makes seawater vaporize. From the point of view, Q_1 reflects the response of atmosphere to SST and the response in just evaporation-SST feedback process. In Eq.(4), g represents non-dimensional parameter having relation with SST. But because the variation of SST is so slow comparing with atmospheric mechanical quantities that g can be as a constant.

Substituting Eq.(4) into Eq.(1), when d =0 we obtain

$$\begin{cases} \dfrac{\partial u}{\partial t} - fv = -\dfrac{\partial \Phi'}{\partial x} \\ fu = -\dfrac{\partial \Phi'}{\partial y} \\ \dfrac{\partial \Phi'}{\partial t} + c_0^{*2}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right) = 0 \end{cases} \tag{5}$$

where

$$c_0^{*2} = (1-g)c_0^2 < c_0^2 \tag{6}$$

Thus, this kind of heating can be taken as diabatic heating with $c_0{}^2$ changing into smaller c_0^{*2}.

By the method of elimination, from Eq.(5) we obtain

$$£_1 v = 0 \tag{7}$$

where

$$£_1 = \frac{\partial}{\partial t}\left(-c_0^{*2}\frac{\partial^2}{\partial y^2}+f^2\right) - b_0 c_0^{*2}\frac{\partial}{\partial x} \tag{8}$$

In middle and high latitudes, $f = f_0$. In low latitudes, $f = b_0 y$ (b_0 is Rossby parameter).

2.2 *The evaporation-wind feedback*

Wind speed (mainly zonal wind speed especially in low latitudes) makes seawater evaporate as vapor, which enters into atmosphere and the released latent heat caused by vapor condensation warms up atmosphere and this process is the so-called evaporation-wind feedback. On this condition Q can be represented as

$$Q_2 = -l\ c_0^2 a\, u \tag{9}$$

where l is a positive constant having relations with the atmospheric density and the vapor density, $-a\,u$ represents evaporation rate, a is a non-dimensional parameter, and when u>0 (west wind), a <0; when u<0 (east wind), a >0.

In general, for only evaporation-wind feedback can't produce low-frequency motions, the evaporation-SST feedback mechanism must be included. When d =0, Eq.(1) becomes

$$\begin{cases} \dfrac{\partial u}{\partial t} - fv = -\dfrac{\partial \Phi'}{\partial x} \\ fu = -\dfrac{\partial \Phi'}{\partial y} \\ \dfrac{\partial \Phi'}{\partial t} + c_0^{*2}\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) = l\, c_0^2 a\, u \end{cases} \tag{10}$$

Using the method of elimination, Eq.(10) can be rewritten as

$$\pounds_2 v = 0 \tag{11}$$

where

$$\pounds_2 \equiv \frac{\partial}{\partial t}\left(-c_0^{*2}\frac{\partial^2}{\partial y^2} + f^2\right) - b_0 c_0^{*2}\frac{\partial}{\partial x} + l\, c_0^2 a\left(f\frac{\partial}{\partial y} + b_0\right) \tag{12}$$

In Eq.(12), $f = f_0$ in middle and high latitudes and $f = b_0 y$ in low latitudes.

2.3 *The CISK mechanism*

Because for the action of the turbulent friction in the boundary layer, A lot of moist air, which converges and lifts to make cumulus convectively develop. Especially in low latitudes, a lot of cumulonimbus masses caused by the released latent heat of condensation makes atmosphere warmer and strengthens the disturbance. Simultaneously upper atmospheric horizontal divergence makes surface pressure descend and strengthens cyclonic circulation. The whole process is named 'CISK'. On this condition Q can be expressed as

$$Q_3 = -h_1 g w_B = -h_1 g\left(\frac{1}{2}h_B z_y\right) = -\frac{h_1 g h_B}{2}\frac{\partial v}{\partial x} = -h c_0^2\frac{\partial v}{\partial x} \tag{13}$$

where w_B if the vertical speed of the top of boundary layer, h_E the standard height of Ekman, $z_g = \dfrac{\partial v}{\partial x}$ represents relative vertical vorticity, h_1 is a constant, $h = \dfrac{h_1 g h_E}{2{c_0}^2} = \dfrac{h_1}{2} * \dfrac{h_E}{H}$ is a non- dimensional parameter which represents the CISK mechanism. Only when w_B or $\dfrac{\partial v}{\partial x} > 0$, h can be a non-zero number. Thus, the equations of barotropic model including the CISK mechanism are written as

$$\begin{cases} \dfrac{\partial u}{\partial t} - fv = -\dfrac{\partial \Phi'}{\partial x} \\ fu = -\dfrac{\partial \Phi'}{\partial y} \\ \dfrac{\partial \Phi'}{\partial t} + c_0^2 \left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} \right) = h\, c_0^2 \dfrac{\partial v}{\partial x} \end{cases} \tag{14}$$

Using the method of elimination, Eq.(14) becomes

$$\pounds_3 v = 0 \tag{15}$$

where

$$\pounds_3 \equiv \frac{\partial}{\partial t}\left(-c_0^2 \frac{\partial^2}{\partial y^2} + f^2 + h c_0^2 \frac{\partial^2}{\partial x \partial y} \right) - b_0 c_0^2 \frac{\partial}{\partial x} - h c_0^2 f \frac{\partial^2}{\partial x^2} \tag{16}$$

In middle and high latitudes, $f = f_0$; In low latitudes, $f = b_0 y$;

Next, we should put emphasis on the analysis of the barotropic diabatic wave in low latitudes.

3 THE DIABATIC WAVES CAUSED BY THE HEATING OF CONVECTIVE CONDENSATION

From Eqs.(7) and (8), the diabatic equation caused by the heating of convective condensation in middle and high latitudes are expressed as

$$\frac{\partial}{\partial t}\left(-c_0^{*2} \frac{\partial^2 v}{\partial y^2} + f_0^{\,2} v \right) - b_0 c_0^{*2} \frac{\partial v}{\partial x} = 0 \tag{17}$$

Using the homogeneous boundary condition of y direction in middle and high latitudes, the solution of Eq.(17) is expressed as

$$v = V e^{i(kx + ly - wt)} \tag{18}$$

where V is the amplitude, k and l represent the wave-numbers in x direction and y direction respectively; w is the angular frequency.

Substituting Eqs.(18) into (17) and the angular frequency is given by

$$w = -\frac{b_0 k}{l^2 + l_0^{*2}} \tag{19}$$

where

$$l_0^{*2} \equiv \frac{f_0^2}{c_0^{*2}} > \frac{f_0^2}{c_0^2} \equiv l_0^2 \tag{20}$$

Clearly, Eq.(19) represents the angular frequency of the long Rossby wave caused by the heating of barotropic convective condensation in middle and high latitudes, but the angular frequency of long Rossby wave which there is no convective condensation is

$$w_0 = -\frac{b_0 k}{l^2 + l_0^2} \tag{21}$$

Because $l_0^{*2} > l_0^2$. Thus, $|w| < |w_0|$. So the diabatic Rossby wave caused by the heating of convective condensation is low-frequency wave with reference to the adiabatic wave.

From Eqs.(7) and (8) we see also that the diabatic wave equation caused by the heating of convective condensation in low latitudes is expressed as

$$\frac{\partial}{\partial t}\left(-c_0^{*2}\frac{\partial^2 v}{\partial y^2} + b_0^2 y^2 v\right) - b_0 c_0^{*2}\frac{\partial v}{\partial x} = 0 \tag{22}$$

where the boundary condition is

$$v\big|_{y\to\pm\infty} \to 0 \tag{23}$$

If we designate the solution of Eq.(22) by

$$v = V(y)e^{i(kx-wt)} \tag{24}$$

Substituting Eq.(24) into Eqs.(22) and (23) yields

$$\begin{cases} \dfrac{d^2V}{dy^2} + \left(-\dfrac{b_0 k}{w} - \dfrac{b_0^2}{c_0^{*2}}\right)V = 0 \\ v\big|_{y\to\pm\infty} \to 0 \end{cases} \tag{25}$$

which is the eigenvalue problem of Weber equation in the form of

$$\frac{d^2 w}{dy^2} - (ax^2 + b)w = 0 \tag{26}$$

Satisfying the boundary condition $w \to 0$ at $x \to \pm\infty$, The eigenvalue and eigenvalue function of Eq.(26) are written as

$$\begin{cases} -\dfrac{b}{\sqrt{a}} = 2m+1 \quad (m = 0, 1, 2, \cdots) \\ w = \exp\left(-\dfrac{\sqrt{a}x^2}{2}\right)H_m\left(\sqrt[4]{a}x\right) \end{cases} \tag{27}$$

Where $H_m(x)$ is the *m*-order Hermite polynomial. Thus, the solution to Eq.(25) is

$$w = -\frac{kc_0^*}{2m+1} \qquad (c_0^* = \sqrt{1-g}\ c_0; m = 0, 1, 2, \cdots) \tag{28}$$

and

$$V(y) = H_m\left(\frac{y}{L_0^*}\right)\exp\left(\frac{y^2}{2L_0^{*2}}\right) \qquad (m = 0, 1, 2, \cdots) \tag{29}$$

where

$$L_0^* \equiv \sqrt{\frac{c_0^*}{b_0}} = \sqrt{1-g}\ L_0 \qquad L_0 \equiv \sqrt{\frac{c_0}{b_0}} \tag{30}$$

Clearly, Eq.(28) represents the angular frequency of ling Rossby wave caused by the heating of barotropic convective condensation in low latitudes. But on this adiabatic condition, the angular frequency of barotropic long Rossby wave in low latitudes is

$$w_0 = -\frac{kc_0}{2m+1} \qquad (m = 0, 1, 2, \cdots) \tag{31}$$

In fact, Eq.(28) becomes (31) when $g = 0$. Here, $|w| < |w_0|$ (as in middle and high latitudes). Thus the heating of convective condensation makes the angular frequency of Rossby wave descend in low latitudes.

Expanding Eq.(28) to $m = -1$, we obtain

$$w = kc_0^* = k\sqrt{1-g}\,c_0 \tag{32}$$

Where w represents the angular frequency of Kelvin wave caused by convective condensation and its value is smaller than the angular frequency of adiabatic Kelvin wave.

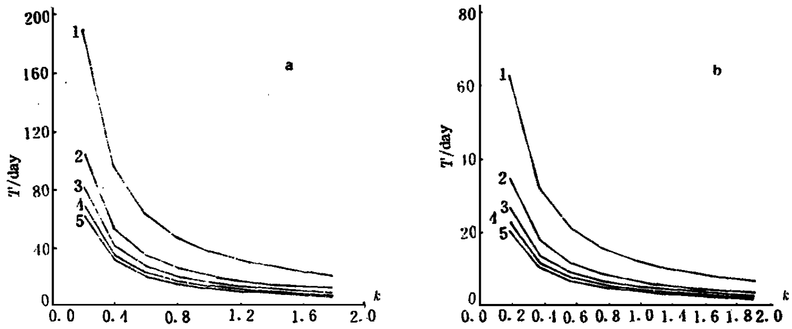

Fig.1 Variation of T with k for different T_s (a) m=1,Rossby wave, (b) m=-1, Kelvin wave

1,2, 3, 4and 5 represent the curves which SST are 24.5, 23.5, 22.5, 21.5 and 20.5

Clearly, when $g > 1$ the angular frequency of Rossby wave and Kelvin wave caused by the heating of convective condensation is a pure imaginary, which can cause the instability of waves. If $g = g(T_s)$, the T_s meeting $g(T_s) = 0$ is called the critical SST. The value of T_s is $25^0 C$ by calculation. Taking $c_0 = 35 m \cdot s^{-1}$, $k = (0.2 \sim 2)\times 10^{-6} m^{-1}$, when m = –1 and 1, $T_s < 25^0 C$, 30-60 day (even longer) diabatic wave is obtained and illustrated by Fig 1.

From the point of view of physical mechanism, the heating of convective condensation or evaporation-SST feedback which can cause atmospheric moisture to rise and make the atmospheric stratification stability descend and make the wave move slowly. Simultaneously, the effects of Q_1 make the regulation of horizontal divergence weaker and make the wave move slowly.

4 THE DIABATIC WAVE CAUSED BY THE HEATING OF CONVECTIVE CONDENSATION AND THE EVAPORATION-WIND FEEDBACK

From Eqs.(11) and (12), the diabatic wave caused by the heating of convective condensation and evaporation-wind feedback in middle and high latitudes is expressed as

$$\frac{\partial}{\partial t}\left(-c_0^{*2}\frac{\partial^2 v}{\partial y^2} + f^2 v\right) - b_0 c_0^{*2}\frac{\partial v}{\partial x} + l\, c_0^2 a\left(f\frac{\partial}{\partial y} + b_0\right)v = 0 \tag{33}$$

Substituting Eq.(18) into Eq.(33) gives

$$w = w_r + iw_i \tag{34}$$

where

$$w_r = \frac{-\left(b_0 k - \frac{la}{\sqrt{1-g}} f_0 l\right)}{l^2 + l_0^{*2}} \qquad w_i = \frac{-\left(\frac{la}{\sqrt{1-g}} b_0\right)}{(l^2 + l_0^*)} \tag{35}$$

Thus, after introducing the evaporation-wind feedback into the heating of convective condensation (or the evaporation-SST feedback), not only the angular frequency of the Rossby wave in middle and high latitudes begins to change but also the wave become unstable. Clearly, when $g < 1$, westerly flow (a <0) makes the wave stronger ($w_i > 0$) but easterly flow (a >0) makes the wave weaker (w_i <0).

From Eqs.(11) and (12), the diabatic equation caused by evaporation-wind feedback and heating of convective condensation in low latitudes is expressed as

$$\frac{\partial}{\partial t}\left(-c_0^{*2}\frac{\partial^2 v}{\partial y^2} + f^2 v\right) - b_0 c_0^{*2}\frac{\partial v}{\partial x} + l\, c_0^2 a b_0\left(y\frac{\partial}{\partial y} + 1\right)v = 0 \tag{36}$$

Substituting Eq.(24) into Eq.(36) gives

$$\frac{d^2V}{dy^2} - \frac{ilab_0}{w(1-g)} y\frac{dV}{dy} + \left[-\frac{b_0 k}{w} - \frac{b_0^2 y^2}{c_0^{*2}} - \frac{ilab_0}{w(1-g)}\right]V = 0 \tag{37}$$

If we designate V by

$$V = \hat{V}\exp\left[\frac{ilab_0 y^2}{4w(1-g)}\right] \tag{38}$$

Eq.(37) becomes

$$\frac{d^2\hat{V}}{dy^2} + \left[-\left(\frac{b_0 k}{w} + \frac{ilab_0}{2w(1-g)}\right) - \left(\frac{b_0^2 y^2}{c_0^{*2}} - \frac{(lab_0)^2}{4w^2(1-g^2)}\right)y^2\right]\hat{V} = 0 \tag{39}$$

which is Weber equation in the form of Eq.(26). Thus, the eigenvalues of Eq.(39) satisfying the boundary condition $w \to 0$ at $x \to \pm\infty$ are

$$\frac{-\left(\frac{b_0 k}{w} + \frac{ilab_0}{2w(1-g)}\right)}{\sqrt{\frac{b_0^2}{c_0^{*2}} - \frac{(lab_0)^2}{4w^2(1-g)^2}}} = 2m+1 \qquad (m = 0, 1, 2, \cdots) \tag{40}$$

Using Eq.(40), we obtain

$$w^2 = A + Bi \tag{41}$$

where

$$A = \frac{c_0^{*2}}{(2m+1)^2}\left[k^2 + \frac{m(m+1)\, l^2 a^2}{(1-g)^2}\right], \qquad B = \frac{lakc_0^2}{(2m+1)^2} \quad (m = 0, 1, 2, \cdots) \tag{42}$$

And the eigenvalue functions of Eq.(39) satisfying the boundary condition $w \to 0$ at $x \to \pm\infty$ are

$$\hat{V}(y) = H_m\left(\frac{y}{L}\right)\exp\left(-\frac{y_2}{2L^2}\right) \qquad (m = 0, 1, 2, \cdots) \tag{43}$$

Thus,

$$V(y) = H_m\left(\frac{y}{L}\right)\exp\left(\frac{i\,l\,a b_0 y^2}{4w^2(1-g)} - \frac{y^2}{L^4}\right) \qquad (m = 0, 1, 2, \cdots) \tag{44}$$

where

$$\frac{1}{L^4} = \frac{b_0^2}{c_0^{*2}} - \frac{(l\,a b_0)^2}{4w^2(1-g)^2} \tag{45}$$

Because $V(y)$ must satisfy the boundary condition $w \to 0$ at $x \to \pm\infty$, we obtain

$$\mathrm{Re}\left[\frac{i\,l\,a b_0}{4w(1-g)} - \frac{1}{2L^2}\right] < 0 \tag{46}$$

Substituting Eqs.(44) and (45) into Eq.(46) gives

$$\mathrm{Re}\left\{\frac{1}{w}\left[k + \frac{(m+1)a\,l}{1-g}i\right]\right\} < 0 \tag{47}$$

Substituting Eq.(34) into Eq.(41) gives

$$w_r^{\,2} - w_i^{\,2} = A, \qquad 2w_r w_i = B \tag{48}$$

Thus,

$$w_r^{\,2} = \frac{1}{2}(A + \sqrt{A^2 - B^2}), \qquad w_i^{\,2} = \frac{1}{2}(-A + \sqrt{A^2 - B^2} \tag{49}$$

$$\begin{aligned} |w|^2 = \sqrt{A^2 - B^2} &= \frac{c_0^2}{(2m+1)^2}\sqrt{\left[\frac{m(m+1)\,l^2 a^2}{(1-g)} + (1-g)k^2\right]^2 - (k\,l\,a)^2} \\ &< \frac{c_0^2}{(2m+1)^2}\left[\frac{m(m+1)\,l^2 a^2}{(1-g)} + (1-g)k^2\right] \end{aligned} \tag{50}$$

Substituting Eq.(34) into Eq.(47) gives

$$kw_r + \frac{(m+1)\,l\,a}{1-g}w_i < 0 \tag{51}$$

Multiplying Eq.(51) with $2w_r^{\,2}$ and $2w_i^{\,2}$ respectively, we obtain

$$\begin{cases} w_r\left[\dfrac{(m+1)\,l\,a}{1-g}I_m(w^2) + k|w|^2 + k\,\mathrm{Re}(w^2)\right] < 0 \\ w_i\left[(m+1)\,l\,a\,(|w|^2 - RE(w^2)) + kI_m(w^2)\right] < 0 \end{cases} \tag{52}$$

Substituting Eqs.(41) and (50) into Eq.(52) gives

$$\begin{cases} \mathrm{sgn}\,w_r = -\mathrm{sgn}(1-g) \\ \mathrm{sgn}\,w_i = -\mathrm{sgn}\,a \end{cases} \tag{53}$$

It is see that when $g<1$, with the westerly flow ($a<0$) the Rossby wave propagates westward ($w_r>0$) and strengthen ($w_i>0$); with the easterly flow ($a<0$) the Rossby wave propagates westward ($w_r<0$) and weaken ($w_i<0$). When m=-1, the Kelvin wave can be analyzed like this.

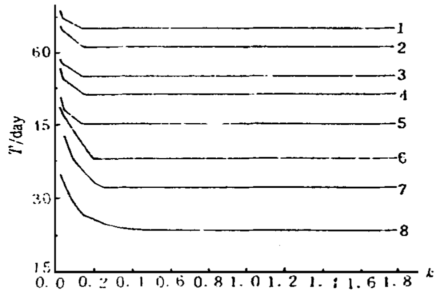

Fig.2 Variation of T with k for different T_s (m=1, Rossby wave) 1, 2, 3, 4, 5, 6, 7and 8 present the curves which SST are 29, 28.5, 28, 27.5, 27, 26.5, 26, 25.5°C

Fig 2 gives the variation of the Rossby wave period with different k and it shows that when $T_s>26^0C$, there are the low-frequency waves whose period is more than 30 days.

Thus, introducing evaporation-wind feedback mechanism into the heating of convective condensation must make the atmospheric moisture rise and make the wave speed slower.

5 THE DIABATIC WAVE CAUSED BY CISK MECHANISM

From Eqs.(15) and (16), the diabatic wave equation caused by the CISK mechanism in middle and high latitudes is expressed as

$$\frac{\partial}{\partial t}\left(-c_0^2\frac{\partial^2 v}{\partial y^2}+f_0{}^2v+hc_0^2\frac{\partial^2 v}{ax\partial y}\right)-b_0c_0^2\frac{\partial v}{\partial x}+hc_0^2 f_0\frac{\partial^2 v}{\partial x^2}=0 \tag{54}$$

Substituting Eq.(18) into Eq.(54) gives

$$w_r=-\frac{b_0k}{l+l_0{}^2-hkl},\qquad w_i=\frac{-hf_0k^2}{l^2+l_0{}^2-hkl} \tag{55}$$

Thus, after introducing the CISK mechanism, the angular frequency of the Rossby wave begins to change and the wave become unstable in middle and high latitudes.

From Eqs.(15) and (16), the diabatic equation caused by the CISK mechanism in low latitudes is expressed as

$$\frac{\partial}{\partial t}\left(-c_0^2\frac{\partial^2 v}{\partial y^2}+f_0{}^2v+hc_0^2\frac{\partial^2 v}{ax\partial y}\right)-b_0c_0^2\frac{\partial v}{\partial x}+hc_0^2 b_0 y\frac{\partial^2 v}{\partial x^2}=0 \tag{56}$$

Substituting Eq.(24) into Eq.(56) gives

$$\frac{d^2V}{dy^2}-ikh\frac{dV}{dy}\left[-\frac{b_0k}{w}--\frac{b_0^2y^2}{c_0^2}-\frac{ihb_0k^2y}{w}\right]V=0 \tag{57}$$

If we designate V by

$$V=\hat{V}\exp\left(\frac{ihky}{2}\right) \tag{58}$$

Equation Eq.(57) becomes

$$\frac{d^2\hat{V}}{dy^2}+\left(-\frac{b_0k}{w}-\frac{b_0{}^2y^2}{c_0^2}-\frac{ihb_0k^2y}{w}+\frac{h^2k^2}{4}\right)\hat{V}=0 \tag{59}$$

If we let

$$x = y + \frac{ihk^2 c_0^2}{2b_0 w} \tag{60}$$

Then Eq.(59) becomes

$$\frac{d^2\hat{V}}{dx^2} + \left(-\frac{b_0 k}{w} - \frac{b_0{}^2 x^2}{c_0^2} - \frac{h^2 k^4 c_0^2}{4w^2} + \frac{h^2 k^2}{4} \right)\hat{V} = 0 \tag{61}$$

In fact, Eq.(61) is the Weber equation in the form of Eq.(26). Thus, the eigenvalues of Eq.(61) satisfying the boundary condition $w \to 0$ at $x \to \pm\infty$ are

$$\frac{-\left(\frac{b_0 k}{w} + \frac{h^2 k^4 c_0^2}{4w^2} - \frac{h^2 k^2}{4} \right)}{\left(\frac{b_0}{c_0^2} \right)} = 2m+1 \qquad (m = 0, 1, 2, \cdots) \tag{62}$$

When there is no CISK mechanism ($h = 0$), Eq.(62) becomes

$$w = -\frac{kc_0}{2m+1} \quad (m = 0, 1, 2, \cdots) .$$

The expression represents the general long Rossby waves $(m = 0, 1, 2, \cdots)$ and the long Kelvin wave (m = -1). When there is the CISK mechanism ($h \neq 0$), from Eq.(62) we obtain

$$\left[\frac{4(2m+1)}{L_0^2} - h^2 k^2 \right] w^2 + 4b_0 kw + h^2 k^4 c_0^2 = 0 \tag{63}$$

thus,

$$w = \frac{-2b_0 k \pm \sqrt{4b^2{}_0 k^2 - h^2 kc_0^2 \left[\frac{4(2m+1)}{L_0^2} - h^2 k^2 \right]}}{\left[\frac{4(2m+1)}{L_0^2} - h^2 k^2 \right]} \tag{64}$$

Because Eq.(64) must degenerate into Eq.(31) when $h = 0$, the '+' in front of radical expression in Eq.(64)is wrong. So

$$w = \frac{-2b_0 k - \sqrt{4b^2{}_0 k^2 - h^2 kc_0^2 \left[\frac{4(2m+1)}{L_0^2} - h^2 k^2 \right]}}{\left[\frac{4(2m+1)}{L_0^2} - h^2 k^2 \right]} \tag{65}$$

The eigenvalue functions of Eq.(60) satisfying the boundary condition $w \to 0$ at $x \to \pm\infty$ are

$$\hat{V} = H_m(x) \exp\left(-\frac{x^2}{2L_0^2} \right) \qquad (m = 0, 1, 2, \cdots) \tag{66}$$

Thus,

$$V = \exp\left[-\frac{\left(y + \dfrac{ihk^2c_0^2}{2bw_0} \right)^2}{2L_0^2} + \frac{ihky}{2} \right] H_m\left(y + \frac{ihk^2c_0^2}{2b_0w} \right) \tag{67}$$

Clearly, Eq.(67) meets the condition of when y$\rightarrow \pm\infty$, $V \rightarrow 0$. Considering the ridical expression of Eq.(65)

$$4b_0^2k^2 - h^2k^4c_0^2\left[\frac{4(2m+1)}{L_0^2} - h^2k^2 \right] = 0 \tag{68}$$

The two kinds of roots must meet

$$h_1^{\ 2} = \frac{2b_0\left[(2m+1) - 2\sqrt{m(m+1)} \right]}{k^2c_0} \tag{69}$$

and

$$h_1^{\ 2} = \frac{2b_0\left[(2m+1) + 2\sqrt{m(m+1)} \right]}{k^2c_0} \tag{70}$$

Considering the denominator of Eq.(67)

$$\frac{4(2m+1)}{L_0^{\ 2}} - h^2k^2 \tag{71}$$

We obtain

$$h_3^{\ 2} = \frac{4(2m+1)b_0}{k^2c_0} \tag{72}$$

Comparing h_1, h_2 *and* h_3 gives

$$0 < h_1^{\ 2} < h_2^{\ 2} < h_3^{\ 2} \tag{73}$$

Then Eq.(65) become

$$\begin{aligned} w &= \frac{\left[\dfrac{2b_0}{k} + c_0\sqrt{(h^2 - h_1^{\ 2})(h^2 - h_2^{\ 2})} \right]}{h^2 - h_3^{\ 2}} \\ &= \frac{\left[\dfrac{2b_0}{k} + \sqrt{\left(\dfrac{2b_0 2}{k} \right)^2 + k^2c_0^{\ 2}h^2(h^2 - h_3^{\ 2})} \right]}{h^2 - h_3^{\ 2}} \end{aligned} \tag{74}$$

Thus we can draw the following conclusions: when $h^2 > h_3^{\ 2}$, the CISK-Rossby wave propagate eastward and are stable; when $h_2^{\ 2} < h^2 < h_3^{\ 2}$, the CISK-Rossby wave propagate eastward and are stable; when $h_1^{\ 2} < h^2 < h_2^{\ 2}$, the CISK-Rossby wave propagate westward and can be unstable; when $0 < h^2 < h_1^{\ 2}$, the CISK-Rossby waves propagate westward and are stable.

Expanding Eq.(65) to m=-1, we obtain

$$w=\frac{2kb_0+\sqrt{4b_0^{\ 2}k^2+h^2k^4c_0^{\ 2}(\frac{4}{L_0^{\ 2}}+k)}}{\frac{4}{L_0^{\ 2}}+h^2k^2} \tag{75}$$

Where w represents the angular frequency of the CISK-Kelvin waves which propagates eastward and is stable. Fig.3 gives the variation of periods of the CISK-Rossby wave ($m=0,1,2,\cdots$) and the CISK-Kelvin wave (m = –1) with different h and shows that these waves can have 30-60 day (even longer) period.

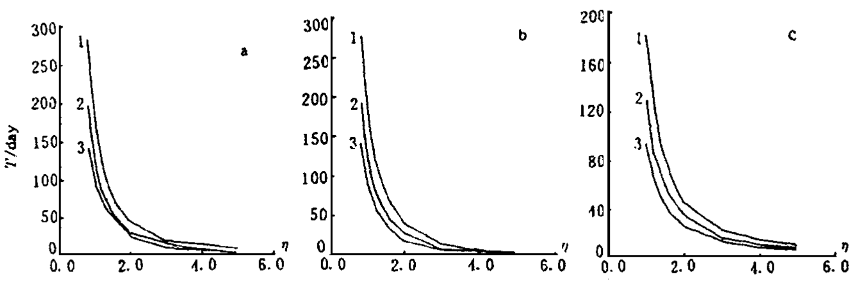

Fig.3 Variation of T with η for different k. (a) m=0 (b) m=1 (c) m = –1; 1, 2 and 3 represent the curves which k are 1.6×10^{-6}/m, 1.8×10^{-6}/m and 2.0×10^{-6}/m

6 REMARKS

Considering the effects of the heating of convective condensation. When $g<1$ (equally $T_s<25^0C$), the Rossby wave and the Kelvin wave are the low-frequency waves but when $g>1$ these waves can be unstable.

The evaporation-wind feedback is an important factor causing the unstable low-frequency waves. Both considering the effects of the heating of convective condensation and the evaporation-wind feedback, the Rossby wave and the Kelvin wave become unstable low-frequency waves and the propagating directions of these waves depend closely on the values of g and a .

The CISK mechanism can produce the low-frequency waves and can cause the instability. The moving and stability of the waves are completely determined by the relative value of h which has three referring values h_1, h_2 *and* h_3. When $h^2>h_3^{\ 2}$, the Rossby waves propagate eastward and are stable; when $h_2^{\ 2}<h^2<h_3^{\ 2}$ and $0<h^2<h_1^{\ 2}$, the Rossby waves propagate westward and are stable; but when $h_1^{\ 2}<h^2<h_2^{\ 2}$, the Rossby waves propagate westward and are unstable. The Kelvin waves merely propagate eastward and all of them are stable.

The diabatic waves are important factors of the long-term climatic evolution.

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