RESEARCH ON REBUILDING OZONE DYNAMIC SYSTEM OF ZONAL AVERAGE OVER THE TROPIC ZONE[1]

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ABSTRACT

The ozone data observed by TOMS in every 5°N are extended into the phase space to describe the characteristics of ozone with phase trace. First of all, the fractional dimension of the ozone layer is calculated. Then, the phase points are regarded as some discrete characteristics solution, and the parameters of mathematical model which describe the time variation of system state are retrieved, so that the nonlinear dynamic system which reflects the short-term variation of zonal average ozone layer over the tropics is rebuilt.

Key words: ozone layer, fractional dimension, state variable, control parameter, dynamic system

I. INTRODUCTION

The issue of evolution, control process and the effect of the ozone layer is one of the important frontier challenges in modern science. At present, many numerical simulations of the ozone layer have been accomplished, but building dynamic models with observed data, which is different from numerical simulation, is another way to rebuild dynamic systems, and a perfect control system isn't needed during the process. It is suggested in the theory of complexity that the evolution of time sequence is the result of the interaction by all key elements (Lin, 1993). The abundant information of all variables that take effect during the evolution of system has been included in the evolution, and not only characteristics and formal process of the system but also the future evolution characteristics have been included in the information. Rebuilding a dynamic system has been given basic theoretic conditions.

Based on the nonlinear theory, the nonlinear characteristics of a system should be kept in the theory and in the method during the process of rebuilding a system from data. Because the system feedback could be reflected in the nonlinear characteristics, the relation among some variables could be obtained. It is significant to reveal the inner relation of a system and to locate the causes that have effect on the system evolution, and get the key to system dynamic characteristics and its control. In the research, zonal average day-to-day total ozone data of TOMS from

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Nov. 1978 to Dec. 1991 in every 5°N (Larko and McPeters, 1992) have been continued in the phase space. The actual nonlinear dynamic system has been built by regarding the state variables of the phase point as the characteristic solution of the tropic ozone model and working out the inverse question.

II. THE DIMENSION IN THE PHASE SPACE OF DYNAMIC SYSTEM

The dynamic system with n state variable $S\{X_i\}$ are partial differential equations, if $\{X_i\}$ are functions of time and space. When one zonal average ozone layer is thought, the state variables are the only function of time t, $X_i = X_i(t)$. The dynamic system is one of ordinary differential equations.

$$\frac{dX_j}{dt} = f_j(\{X_i\},\{p_k\}) \tag{1}$$

where p_k are control parameters of the system characteristics and the outward environment. j=1,2,······, n is the number of equations. f_j is the nonlinear function of state variable $\{X_i\}$.

The evolution of day-to-day 5°N total ozone including 4800 points is described in Fig.1. The annual and quasi-biennial oscillation of total ozone and its erratic daily variation can be observed in the figure. The maximum can be reached in July, and the minimum in January. The maximum, minimum and average values are 293, 239 and 265 Du, respectively, with the maximum range of 54 Du. Compared with the data from other latitudes, the total ozone here is minimum and the daily. seasonal and annual oscillation are not obvious but complex.

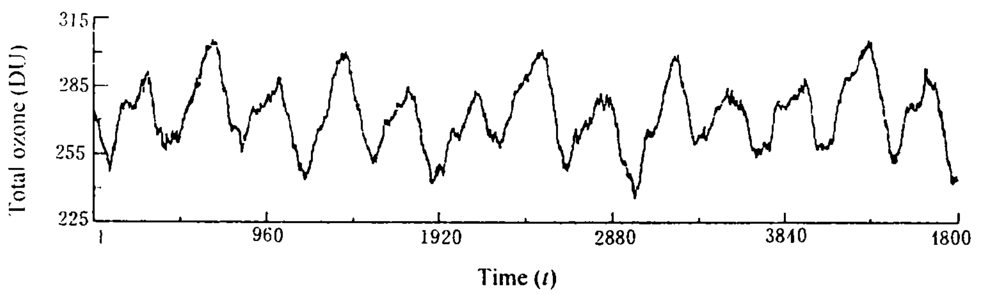

Fig.1. The time sequence of day-to-day zonal total ozone every 5°N from Nov. 1978 to Dec.1991.

A photochemical theory about the formation and the scavenging of ozone in the stratosphere has been given by Chapman (Tang et al., 1990) In the theory the formation reactions of ozone are caused by the effect between ultraviolet less than 242 nm and oxygen molecule.

$$O_2 + h\gamma(\lambda \leq 242\text{nm}) \rightarrow 2O(^3P) \tag{2}$$

$$2O(^3P) + 2O_2 + M \rightarrow 2O_3 + 2M \tag{3}$$

with total reaction $3O_2 + h\gamma \rightarrow 2O_3$.

The scavenging reactions are caused by the photochemical dissociation. which is resulted from the visible and ultraviolet light.

$$O_3 + h\gamma \rightarrow O_2 + O(^3P) \tag{4}$$

$$\underline{O_3 + O(^3P) \rightarrow 2O_2} \tag{5}$$

with total reaction $2O_3 + h\gamma \rightarrow 3O_2$.

Because the oxygen atom and the oxygen molecule can form ozone molecule according the Eq. (4), ozone can be partly cleared, and at the same time a lot of solar radiation will be absorbed by ozone.

$$O_3 + h\gamma(210\,\text{nm} \le \lambda \le 310\,\text{nm}) \rightarrow O_2(^1\Delta) + O(^1D)$$

$$O(^1D) + M \rightarrow O(^3P) + M \tag{6}$$

$$O_3 + h\gamma(400\text{nm} \le \lambda \le 1140\text{nm}) \rightarrow O_2(^3\Sigma) + O(^3P)$$

In addition, those natural or manmade fine substances (organic matter, hydrogenous, nitric and chloric matter) also have important effect on the ozone loss.

Because the formation and the loss of ozone are mostly caused by photochemical dissociation, the reactive speed of equations from (2) to (5) can be affected by the solar radiation and can vary with latitude and altitude. For this reason, the ozone in the upper stratosphere is produced rapidly, and its main formation region is from 10°S to 35°N in Jun., 38°S—12°N in Dec., and the region in 10°S—12°N is always the source of ozone. Based on the theory of photochemistry, the maximum and the minimum of ozone should appear in the tropical and in the polar areas, but the almost opposite situation has been found in observation. The longitudinal gradient of ozone can't be explained by the photochemical theory (Wang, 1991). The redistribution of ozone will be caused by air motion. The produced ozone in low latitude by photochemical reaction can be transferred to the high latitude area, and it can undulate with the modification of the general circulation. The distribution of the ozone layer is complicated by the general circulation and the photochemical reaction. The true result will not be got if only one-dimensional variables are used in the research of ozone. In practice, we can only get the ozone data from single observation, so it is necessary to rebuild ozone dynamic system by retrieving one dimensional sequence in m dimensional space and by getting necessary information of dynamical characteristics and attractor's structure (Wang, Xie and Qiu et al., 1997).

In order to use the phase space with n variables to study the characteristics of the attractor in the ozone layer as well as retain all information of the system evolution, the method of zero-lag correlation-time has been used. Firstly, the expression t=102 as the first zero point of the lag correlation function will be worked out from the data sequence in Fig.1. Secondly, t=102 will be regarded as the best estimate value of the time-lag τ, Thirdly, the m-dimensional space will be rebuilt with the coordinate lag, and the asymptotic characteristics of ozone can be described by the state trace in Fig.2. Finally, the evolution of the system and the minimum dimensions D_2 of its final will be derived. While the dimension m is added, the structure of system state keeps no addition, so the information of n=INT(D_2 +1) variables which are needed to describe system can be obtained.

Based on the calculation of correlation dimension (Liu et al., 1993), the curve of $\ln C_m(r)$ and $\ln r$ has been given in Fig.3. The correlation dimension has been estimated by the slope of the

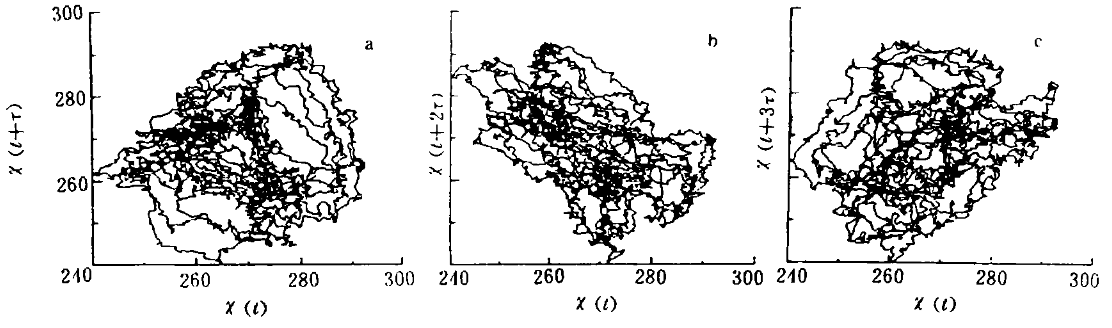

Fig.2. Two dimensional phase portraits for 5°N zonal average day-to-day total ozone (Time lag $\tau = 102$).

linear part. It can be seen from Fig.4 that the correlation dimension is D_2=3.18 when m equals 8. The ozone dynamic system to be built needs 4 variables at least. In order to research the perturbation of ozone layer, the original sequence of Fig.1 has been dealt into the anomaly sequence, and the anomaly sequence has been continued into 4 dimension with the lag parameter τ=102. The phase type distribution is obtained by transferring the original sequence according to its average value, so compared with Fig.2, the structure of system state is the same, but the position are different. Thus, the state variables in four dimensions' space are:

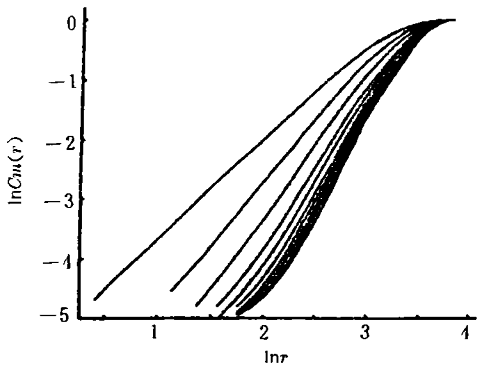

Fig.3. The relation of $\ln C_m(r)$ and $\ln r$ of the area ozonosphere series.

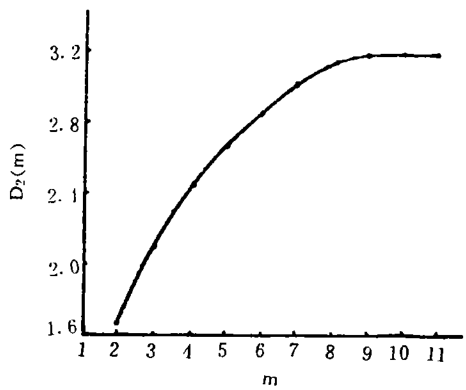

Fig.4. The relation of attractor's dimension D_2(m) and the phase space's dimension.

$$X=\left\{X_1(t),\ X_2(t),\ X_3(t),\ X_4(t)\right\}=\left\{x(t),\ x(t+t),\ x(t+2t),\ x(t+3t)\right\} \quad (7)$$

where the X_1, X_2, X_3, X_4 are state components of the anomaly sequence in the different time interval t=1, ⋯, 4494; $t+\tau$=103, ⋯, 4596; $t+2\tau$=205, ⋯, 4698; $t+3\tau$=307, ⋯, 4800.

III. THE DYNAMIC CONTROL PARAMETERS OF SYSTEM

It is supposed that the evolution of state variables can be described by Eq. (1), and the ($f_i\{X_i\},\{p_k\}$) is a polynomial function with linear and nonlinear secondary items. The dynamic equations are:

$$\left.\begin{aligned}
\frac{dX_1}{dt} &= a_1X_1+a_2X_2+a_3X_3+a_4X_4+a_5X_1^2+a_6X_2^2+a_7X_3^2+a_8X_4^2+a_9X_1X_2\\
&\quad +a_{10}X_1X_3+a_{11}X_1X_4+a_{12}X_2X_3+a_{13}X_2X_4+a_{14}X_3X_4\\
\frac{dX_2}{dt} &= b_1X_1+b_2X_2+b_3X_3+b_4X_4+b_5X_1^2+b_6X_2^2+b_7X_3^2+b_8X_4^2+b_9X_1X_2\\
&\quad +b_{10}X_1X_3+b_{11}X_1X_4+b_{12}X_2X_3+b_{13}X_2X_4+b_{14}X_3X_4\\
\frac{dX_3}{dt} &= c_1X_1+c_2X_2+c_3X_3+c_4X_4+c_5X_1^2+c_6X_2^2+c_7X_3^2+c_8X_4^2+c_9X_1X_2\\
&\quad +c_{10}X_1X_3+c_{11}X_1X_4+c_{12}X_2X_3+c_{13}X_2X_4+c_{14}X_3X_4\\
\frac{dX_4}{dt} &= d_1X_1+d_2X_2+d_3X_3+d_4X_4+d_5X_1^2+d_6X_2^2+d_7X_3^2+d_8X_4^2+d_9X_1X_2\\
&\quad +d_{10}X_1X_3+d_{11}X_1X_4+d_{12}X_2X_3+d_{13}X_2X_4+d_{14}X_3X_4
\end{aligned}\right\} \tag{8}$$

A sort of algebraic equation have been got by separating the time differential quotient in the Eq.(8). Except the control parameters $\{a_k\}$, $\{b_k\}$, $\{c_k\}$, $\{d_k\}$, the other variables have their temporal functions in Eq.(7). The parameters have been regarded as the assemble of the characteristics solution in different time. The parameters have been worked out based on the theory and the method of inversion (Chou,1986; Yan and Peng, 1993), and the time-varying dynamic system of the ozone evolution can be got. While solving the inversion question of the state variables in No. j equation, each item in the polynomial function of ($f_i\{X_i\},\{p_k\}$) which correspond to each state variable comes into the coefficient control parameters, and the time differential d are got from state variable sequence X_i.

$$\left[f_i(\{X_i(l+1)\},\{p_k\})\right]_l = \sum_{k=1}^{q} G_{lk}p_k \tag{9}$$

$$\frac{X_i(l+2)-X_i(l)}{2\Delta t} = d_l \tag{10}$$

where the $\{X_i(l+1)\}$ stands for No. l+1 state vector. The q=14 is the number of items in the polynomial function. The Δt=1 is the time interval the following is got.

$$d_l = \sum_{k=1}^{q} G_{lk}p_k \qquad (l=1,2,\cdots,M) \tag{11}$$

where the M=N-2, N=4494 are the lengths of the state variable sequence. M equations can be constructed with Eq.(7) and the state variable sequence X_i and they are given in matrix form:

$$\boldsymbol{D}=\boldsymbol{GP} \tag{12}$$

where $\boldsymbol{D},\boldsymbol{G},\boldsymbol{P}$ are the

$$\boldsymbol{D}=\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_l \\ \vdots \\ d_M \end{bmatrix}=\begin{bmatrix} \frac{X_l(3)-X_l(1)}{2\Delta t} \\ \frac{X_l(4)-X_l(2)}{2\Delta t} \\ \vdots \\ \frac{X_j(l+2)-X_l(l)}{2\Delta t} \\ \vdots \\ \frac{X_l(N)-X_l(N-2)}{2\Delta t} \end{bmatrix}; \quad \boldsymbol{G}=\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1q} \\ G_{21} & G_{22} & \cdots & G_{2q} \\ \vdots & \vdots & \cdots & \vdots \\ G_{l1} & G_{l2} & \cdots & G_{lq} \\ \vdots & \vdots & \cdots & \vdots \\ G_{M1} & G_{M2} & \cdots & G_{Mq} \end{bmatrix}; \quad \boldsymbol{P}=\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \\ \vdots \\ p_q \end{bmatrix}$$

Firstly, the vector $\boldsymbol{D}$ is determined by the X_l sequence. Secondly, the q parameters are constructed by the state variables of the $l+1$ phase point in the X_l sequence. Thirdly, a $M\times q$ matrix G has been comprised of $M(2\sim N-1)$ state vectors. $\boldsymbol{P}$ is an unknown control parameter matrix.

$\boldsymbol{P}$ can be got from the contradictory equation of equation (11) by proper method. A least square method has been used in order to keep the square of residual and S minimum

$$S=(\boldsymbol{D}\text{-}\boldsymbol{GP})^{\mathrm{T}}(\boldsymbol{D}\text{-}\boldsymbol{GP}) \quad (13)$$

The superscript T stands for the transpose of matrix, and the following equation has been got:

$$\boldsymbol{G}^{\mathrm{T}}\boldsymbol{GP}=\boldsymbol{G}^{\mathrm{T}}\boldsymbol{D} \quad (14)$$

If the matrix $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}$ isn't singular, its counter matrix exists. The parameter matrix can be got:

$$\boldsymbol{P}=(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G})^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{D} \quad (15)$$

If the matrix $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}$ is singular or mostly singular, the Eq. (15) is alergic to the fine error of $\boldsymbol{D}$. and it will cause the large difference of $\boldsymbol{P}$ so the true parameter matrix cannot be got. The retrieval method has been used to get control parameter truly. The $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}$ is a q step real symmetric matrix, so its characteristics values have been worked out, supposing $|\lambda_1|\geq|\lambda_2|\geq\cdots\geq|\lambda_L|\geq|\lambda_q|$ and q characteristics vectors of linear non-interactive have been got. If q-L characteristics values are zero, or almost zero a $q\times L$ matrix $\boldsymbol{U}$ can be comprised of L nonzero normal characteristics vectors which correspond to the L nonzero characteristics values. Each line of the matrix $\boldsymbol{U}$ U_i(l=1, 2, $\cdots$, L) is a characteristics vector which corresponds to λ_i. Then, non–zero characteristics values have been used to get matrix $V_i=GU_i/\lambda_i$ of V. Where λ_1, λ_2,$\cdots\cdots$ λ_L. are the matrix elements of diagonal matrix $\boldsymbol{\Lambda}$, $\boldsymbol{H}=\boldsymbol{U}\boldsymbol{\Lambda}^{-1}\boldsymbol{V}^{\mathrm{T}}$ can be got by the relation of $\boldsymbol{U}$ and $\boldsymbol{V}$. Finally, a reliable parameter matrix can be got:

$$\boldsymbol{P}=\boldsymbol{HD} \quad (16)$$

The control parameters of other equations in Eq.(8) can also be worked out with the same method.

IV. THE DYNAMIC SYSTEM

Ozone layer is a complex variable system, so the variation of environment can affect its characteristics, cause the adjustment of its inner characteristics, and make it responsive by its nonlinear feedback and instability. The response is controlled by the parameters that describe the characteristics of system and the outward environment. In order to review the function of each parameter to the evolution of system, the relative variances R_k of each item in the equation have been calculated.

$$R_k = \frac{1}{M}\sum_{l=1}^{M}[T_k^2/(\sum_{k=1}^{q}T_k)]_l \tag{17}$$

The T_k stands for the right items in Eq.(11). The results of inversion and the effect of relative variances of corresponding items have been filled in Table 1. The importance of each item in the Eq.(8) can be distinguished from the value of its relative variance. The method, in which items having important effects on the evolution of system have been kept, has been adopted in order to determine the control and the inversion strength of state variables. The method can stress the main points of the question and can reveal the main mechanism during the process. The items in which R_k are less than 6.0×10^{-2} have been removed, so the dynamic equations of ozone system are built.

Table.1. The control parameters and the effect of the variances of each item in dynamic equations.

Item	$\frac{dX_1}{dt}$		$\frac{dX_2}{dt}$		$\frac{dX_3}{dt}$		$\frac{dX_4}{dt}$	
(K)	a_k	R_k	b_k	R_k	c_k	R_k	d_k	R_k
1	2.211×10^{-3}	3.970×10^{-2}	8.942×10^{-3}	4.316×10^{-1}	-4.290×10^{-3}	1.152×10^{-1}	-5.256×10^{-4}	2.320×10^{-3}
2	-1.139×10^{-2}	5.016×10^{-1}	-1.058×10^{-3}	1.602×10^{-2}	1.063×10^{-2}	3.819×10^{-1}	-6.227×10^{-3}	2.111×10^{-1}
3	5.355×10^{-3}	1.778×10^{-1}	-7.354×10^{-3}	3.150×10^{-1}	-2.248×10^{-3}	3.078×10^{-2}	1.223×10^{-2}	4.720×10^{-1}
4	2.397×10^{-3}	4.820×10^{-2}	3.426×10^{-4}	1.299×10^{-3}	-6.498×10^{-3}	1.847×10^{-1}	-8.366×10^{-4}	7.110×10^{-3}
5	-2.127×10^{-5}	1.002×10^{-3}	2.469×10^{-5}	6.530×10^{-4}	2.871×10^{-6}	1.362×10^{-5}	1.586×10^{-4}	5.494×10^{-2}
6	-1.242×10^{-4}	1.094×10^{-2}	4.495×10^{-5}	8.062×10^{-3}	-1.360×10^{-5}	1.229×10^{-4}	-7.415×10^{-5}	6.262×10^{-3}
7	-1.914×10^{-5}	5.393×10^{-4}	-1.450×10^{-4}	2.429×10^{-2}	2.383×10^{-4}	8.321×10^{-2}	-2.820×10^{-4}	4.246×10^{-2}
8	-8.319×10^{-5}	1.653×10^{-2}	1.445×10^{-4}	6.067×10^{-2}	3.999×10^{-5}	1.641×10^{-3}	-2.784×10^{-5}	2.422×10^{-3}
9	9.442×10^{-5}	3.451×10^{-3}	-1.483×10^{-4}	1.231×10^{-2}	1.711×10^{-4}	9.726×10^{-3}	-2.411×10^{-4}	2.959×10^{-2}
10	-4.543×10^{-4}	1.093×10^{-1}	3.446×10^{-4}	4.384×10^{-2}	-7.301×10^{-5}	3.129×10^{-3}	-3.105×10^{-5}	3.363×10^{-4}
11	3.242×10^{-4}	6.729×10^{-2}	-8.842×10^{-5}	4.045×10^{-3}	-4.860×10^{-4}	8.530×10^{-2}	2.178×10^{-4}	3.809×10^{-2}
12	2.309×10^{-4}	1.605×10^{-2}	-1.654×10^{-4}	1.670×10^{-2}	-3.502×10^{-4}	3.525×10^{-2}	5.205×10^{-4}	6.229×10^{-2}
13	1.268×10^{-4}	6.966×10^{-3}	-2.397×10^{-4}	6.391×10^{-2}	4.328×10^{-4}	5.196×10^{-2}	-3.359×10^{-4}	6.600×10^{-2}
14	-3.345×10^{-6}	6.513×10^{-4}	5.296×10^{-5}	1.585×10^{-3}	-2.132×10^{-4}	1.702×10^{-2}	1.220×10^{-4}	5.112×10^{-3}

The inner main characteristics and the relation of main variables of ozone layer can be reflected with the negative or positive response. The dynamic equations describing 5°N zonal ozone layer have been written as follows:

$$\left.\begin{aligned}
\frac{dX_1}{dt} &= -0.01139X_2 + 0.005355X_3 - 0.0004543X_1X_3 + 0.0003242X_1.X_4 \\
\frac{dX_2}{dt} &= 0.008942X_1 - 0.007354X_3 + 0.0001445X_4^2 - 0.0002397X_2X_4 \\
\frac{dX_3}{dt} &= -0.00429X_1 + 0.01063X_2 - 0.006498X_4 + 0.0002383X_3^2 - 0.000486X_1X_4 \\
\frac{dX_4}{dt} &= -0.006227X_2 + 0.01223X_3 + 0.0005205X_2X_3 - 0.0003359X_2X_4
\end{aligned}\right\} \tag{18}$$

An experiment of single step prediction with Eq.(18) has been finished for the evolution of the last 400 phase points constructed by state variables. The comparison of the evolution between the anomaly sequences from model and observation at t=4400, ···, 4800 have been given in

Fig.5. Based on the comparison, the inspection of the dynamic system to model the day-to-day variation of the tropical ozone has been finished, and the error analysis of the model has been filled in Table 2.

Table.2. The error analysis of modeling values and observational anomaly sequence.

Average absolute error	Average relative error(%)	Root-mean-square error	Correlation coefficient
0.2935	7.1341	0.3332	0.9981

It is obvious in Fig.5 and Table 2 that the modeling sequence is good at describing the variation of the ozone anomaly sequence, and their relative coefficient is 0.9981. It is significant that one dimension time sequence is continued in the phase space and dynamic system is retrieved by using the state variables of the phase points. It can describe the nonlinear characteristics in the short-range evolution of ozone layer system.

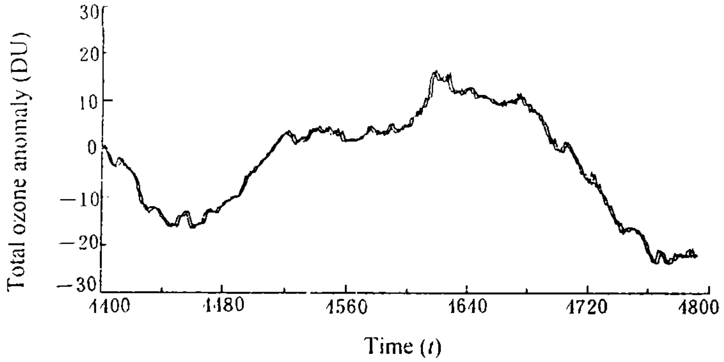

Fig.5. The comparison of evolution between actual anomaly sequence and modeling anomaly sequence.

V. CONCLUDING REMARKS

a. The 5° day-to-day zonal total ozone data has been continued in phase space, and it has been found that 4 variables are needed in describing the evolution of ozone layer. Though the variables are unknown, the time sequence has included the trace of the variables that control the characteristics of system. It is helpful to research the characteristics of ozone layer that the inverse dynamic system is used to describe the evolution of the state.

b. How to describe ozone layer is really an important and difficult problem, because the inner relation and the effect of ozone are not fully known. It is an effective way to recognize the dynamic characteristics of ozone layer that the dynamic can be rebuilt actually, if we take the one dimension time sequence of ozone as known information, and examine its inner dynamic characteristics to make the dynamic function and control parameters determined by the data.

c. Because ozone layer has its nonlinear response effect, the processes can be affected by the results. As a basic experiment of chaotic sequence, the nonlinear and linear square dynamic perturbative system of 5° zonal day-to-day total ozone have been retrieved. Based on the error analysis, the results of short-range prediction are in accordance with that of the perturbation sequence of observation. The results have reflected the complicate coupling relation and the evolution process of the ozone system and it is significant to comprehend the control reasons deeply.

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