ANALYTIC STUDY ON THE MECHANISM FOR LOW FREQUENCY TELECONNECTION AND HORIZONTAL PROPAGATION IN THE ATMOSPHERE

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Received 13 October 1997, accepted 29 December 1997

ABSTRACT

(1) Assuming that there is a zero-divergent layer in mid-troposphere, a diabatic and quasi-geostrophic equation set (bearing the format of that for barotropic vorticity) is derived by vertical integration of the thermodynamics equation and approximate description of anomalous tropospheric heating field in a simultaneous equation. (2) Discussing the spectral mean and group velocities, it is proved that the enclosed centers of the climatically mean geopotential field are no other than the atmospheric low-frequency oscillators in the mid- and higher- latitudes. They have reversed variation of phase on two sides, with the energy supplied by the positive feedback of condensation by CISK or the sensible heat of sea temperature. (3) A formula is derived for low-frequency teleconnecting rays, which are shown to cross the streamlines southward in the northerly or northward in the southerly and to change direction of movement at the bottom of troughs or the top of ridges. Comparing to the great circle argument, the theoretic results above are more reasonable in explaining the observed low-frequency teleconnection in the Northern Hemisphere.

Key words: low-frequency teleconnection, horizontal propagation, wave ray formula

Ⅰ. INTRODUCTION

Since the association of low-frequency teleconnection with the Rossby wave dispersion by Hoskins and Karoly (1981), more work is documented on the diagnostic analysis and numerical simulation but less on analytic discussions. Later on, Hoskins and Ambrizzi (1993) elaborate by assuming the basic airflow to be inhomogeneous, but fail to answer questions like how the low-frequency oscillation is generated, the role of the heating field, the location of static teleconnection and detailed relationship between the track and flow field in the teleconnection, etc. One of the useful points about the analytic approach is that it both pinpoints on the true nature of the issue and provides ready and workable ways for solution. The addressing aspect of the current work is the diabatic 250 hPa vorticity equations, as the barotropic non-divergence equations are proved (details referred to Hoskins et al.) to have considerable contribution in studies of the low-frequency oscillation in the atmosphere and the atmospheric behavior at 250 hPa is the closest to the equations.

Ⅱ. BASIC FORMULA AND EQUATIONS

1. *Vertical motion*

With the assumption that there is a layer of non-divergence p_N in the middle troposphere, the continuity equation is integrated in coordinates (x, y, p, t) and that the vertical motion is zero towards the lower boundary of the tropopause p_T (p_T=250 hPa), and setting $\varepsilon \leqslant 1$, in which ε is a coefficient to be determined, an approximate expression is obtained:

$$\omega(p)-\omega(p_N)=-\int_{p_N}^{p}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\mathrm{d}p \cong \varepsilon(p_N-p)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_{p_T} \quad (1)$$

where $\omega(p)$ is the vertical motion $\frac{\mathrm{d}p}{\mathrm{d}t}$ on the isobaric surface p. Due to the assumption of $\omega(p_T)=0$, we have the following expression at $p=p_N$:

$$\omega(p_N)\cong-\varepsilon(p_N-p_T)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_{p=p_T} \quad (2)$$

2. *Thermodynamics equations in vertically integrated baroclinic troposphere*

Setting a flat underlying surface where $\omega_S=0$ at $p=p_S$, and integrating the thermodynamics equations on the isobaric surface in the vertical, approximate expressions are obtained with $\omega(p_N)$ removed by applying Eq.(2) following the theorem of integral medium:

$$\frac{\partial\Phi_T}{\partial t}+u_*\frac{\partial\Phi_T}{\partial x}+v_*\frac{\partial\Phi_T}{\partial y}+c_a^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_{p_T}=\tilde{Q} \quad (3)$$

$$\tilde{Q}=-\frac{R}{c_p}\int_{p}^{p_T}\frac{\dot{Q}}{p}\mathrm{d}p=\frac{g}{c_pT_*}\int_0^H\dot{Q}\mathrm{d}z \quad (4)$$

$$c_a^2=\varepsilon\frac{R(\gamma_a-\gamma)}{2g}RT_*\frac{(p_s-p_T)(p_N-p_T)}{p_Sp_T} \quad (5)$$

where Φ_T is the gravity potential on the isobaric surface p_T while the change of potential Φ_S on the surface p_S is neglected. $\dot{Q}$ is the heating rate for unit mass of the atmosphere, H the altitude corresponding to the lower boundary of the tropopause p_T (setting H=11000 m). u_*, v_*, and T_* are the mid-values of u, v, and T in vertical integration. Here, as p_S =1000 hPa, p_S =550 hPa, γ=0.65℃/100 m, and ε=0.5, then $c_a \cong 42.5$ m/s.

Eq.(3) resembles the continuity equations of the shallow water barotropic model in shape but differs from it in that Φ_T and divergence take values at 250 hPa while u_* and v_* are close to the vertical mean over the troposphere. $\dot{Q}$ is the product of tropospheric mean heating rate multiplied by mechanical efficiency $\eta_0=\frac{g}{c_p}T_*$, $\eta_0\cong0.37$.

3. *Vorticity equations*

$$\frac{\mathrm{d}f+\zeta}{\mathrm{d}t} = -(f+\zeta)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\mu\nabla^2\zeta \tag{6}$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vorticity on the isobaric surface, f is the Coriolis force and μ the coefficient of dynamic friction.

III. ANOMALOUS HEATING FIELD

It is difficult to give analytic description of heating field $\dot{Q}$ but much easier if the attempt is focused on the anomalous heating field $\widetilde{Q}' = \widetilde{Q} - \widetilde{Q}_{\mathrm{m}}$. $\widetilde{Q}_{\mathrm{m}}$ is the climatological mean of $\widetilde{Q}$ (including seasonal variation, same below). Eq.(4) is based to obtain

$$\widetilde{Q}' = A\left(\frac{gH}{c_pT_*}\right)S' + A\left(\frac{gH}{c_pT_*}\right)\left(4\sigma T_S^3 T_S{}' - 8a\sigma T_*^3 T_*'\right)$$

$$+ Q_3' + \eta_4 \frac{gL}{c_pT_*}\int_{h_T}^{h_B} W'\frac{\partial q_S}{\partial z}\mathrm{d}z + \widetilde{Q}_L' \tag{7}$$

where the first term of Eq.(7) is the changes in solar shortwave radiation, in which $S'=0$ is established if the solar constant and atmospheric elements keep unchanged in approximation. In the meantime, the sun-rays reflected off by clouds are thought to be offset substantially by those that are additionally absorbed due to the very existence of clouds. The second term is the anomalies of longwave radiation budget, in which T_S is the temperature of the underlying (land or oceanic) surface, σ the Boltzmann's constant, a the radiation capability in the atmosphere. $A=10^{-7}\mathrm{m}^2/g$, which is the ratio of unit horizontal sectional area to the mass of the air column passing through the area. The third term, Q_3', is the anomalies of sensible heating and the fourth term is the anomalies of latent heat from condensation, in which h_B is the altitude at which condensation takes place, h_T the height of cloud top, W' the anomalies of vertical motion in the z coordinates, q_S the saturation specific humidity and η_4 the condensation efficiency. $\widetilde{Q}_l'$ is the anomalies for other heating terms. For land surface, the anomaly term Q_3' may be removed in some way for the exchange of sensible heat and the sensible heat anomaly transferred to the atmosphere from the ocean can be expressed by

$$Q_3' = c_d c_p U_0 (T_S' - T_*') \tag{8}$$

where $c_d = 10^{-3}$, and ρ_0 and U_0 are the air density and wind speed at the underlying surface, with climatological mean is taken for the latter.

An assumption is contained here that temperature anomaly is identical with mean tropospheric temperature on the underlying surface. Introducing the thermodynamics equations for the surface and including the radiation, sensible heating and heat consumption budget by evaporation, the anomalous equation may be written as

$$c_S \frac{dT_S}{dt} = -\eta\left(\frac{\eta_4}{A}\frac{gL}{c_p T}\int_{h_T}^{h_b} W'\frac{\partial q_s}{\partial z}dz\right) - Q'_E$$

$$+ (4a\sigma T_*^3 T'_* - 4\sigma T_S^3 T'_S) - \frac{Q'_3}{A} \tag{9}$$

where c_S is the thermal capacity in unit area of the underlying surface. In Eq.(9), the first term to the right of the equality sign is the anomalies of direct radiation magnitude that is received on the underlying surface arisen from clouds and precipitation. With the assumption that it is in positive proportion with condensation latent heat and the coefficient of proportionality is η, it can be inferred that η=(20-3-4)/23=0.565, because 23% of the solar radiation energy input to the atmosphere is changed into condensation latent heat while 20% of it is reflected by clouds, which in reverse absorb 3% of the solar energy, a fact that is balanced out by unavailability of reflective loss (4%) for cloud-cover free underlying surface, as confirmed by well-known satellite observations.

The third term on the right hand side of Eq.(9) shows the budget for longwave radiation of the underlying surface. Q'_E is the heat anomaly consumed in evaporation. Q'_3 is the anomaly of sensible heat exchange as given in Eq.(8). The integration can be written for Eqs.(8) and Eq.(9):

$$\int_{h_T}^{h_b} W'\frac{\partial q_s}{\partial z} = \int_{h_T}^{h_b} W' dq_s = W'_*(q_{sB} - q_{sT}) \tag{10}$$

$$W'_* = \frac{W'_b W_N}{2}\frac{h_N - h_b}{H - h_b} + \frac{W'_N}{2}\frac{H - h_N}{H - h_b} \cong \frac{1}{2}(W'_N + 0.4W'_b) \tag{11}$$

where W'_* is the integral mean of W', whose expression is shown in Eq.(11). The approximate expression to the right takes the approximation when h_N=5000 m ($p_N \cong 550$ hPa), h_b=1000 m, H=11000 m. On the other hand, $W'_N = -(RT_*/g)(\omega_N/p_N)$ and q_{sB} and q_{sT} are saturation specific humidity for the base and top of clouds. (q_{sB}-q_{sT}) varies between zero and q_{sB}, because the minimum of q_{sT}, q_{sH}, is the saturation specific humidity at the height H, which is so small that it is comparable to q_{sB}. Consequently, (q_{sB}-q_{sT}) in Eq.(10) is replaceable with the simple mean $q_{sB}/2$ while $q_{sB} \cong q_S$, in which q_s is the specific humidity for the underlying surface (at the height of the screen). For simplicity, q_s only takes the climatological mean (including mean changes with season).

W'_B is vertical motion at the cloud base, which can be substituted with the pumping velocity at the top of the frictional layer (generally with $W_B = b\zeta_0$, ζ_0 is the quasi-geostrophic vorticity there). It is noted that on the zero-divergence layer (dζ/dt)=0 and the sign changes to the opposite on the upper and lower troposphere, as inferred from Eq.(6), a vorticity equation, with the condition that μ=0 and generally observes the movement along the zonal direction. Opposite signs are therefore set for ζ'_0 and ζ'_T, the anomalous vorticity on the layer p_T. Assuming that

$$W'_{\mathrm{B}} = -b_1\zeta_{\mathrm{T}} \tag{12}$$

where b_1 is an undetermined coefficient and has dimension of length just as b. By substituting Eq.(12) into Eq.(11) and then to Eq.(10), and substituting Eq.(10) to Eqs.(8) and (9), which are made simultaneous, with the sensible exchange term Q'_3 eliminated between the land surface and the atmosphere, the anomalous heating field is then rewritten as

$$\begin{aligned}\widetilde{Q}' = & -4\eta_0 a\sigma A T_*^3 T_*' + \delta\left[4\eta_0\sigma A(T_*^3 T_*' - aAT_*^3 T') + c_d c_p \rho_0 U_0 (T_{\mathrm{S}}' - T_*')\right] \\ & -\left[1-(1-\delta)\eta_0\eta\right]0.4\eta_0 \frac{b_1}{H}\frac{q_{s\mathrm{B}} - q_{s\mathrm{T}}}{4}\eta_4 L\zeta_{\mathrm{T}}' \\ & -\left[1-(1-\delta)\eta_0\eta\right]\frac{R}{c_p}\frac{q_{s\mathrm{B}} - q_{s\mathrm{T}}}{8}\frac{p_{\mathrm{N}} - p_{\mathrm{T}}}{p_{\mathrm{N}}}\eta_4 L\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right)_{p_1}\end{aligned} \tag{13}$$

where $\delta=1$ for the oceanic surface and $\delta=0$ for the land surface. The term c_{S} (small in magnitude) and heat consumption by evaporation are negligible if it is the latter case. Under the condition, there will be no T_{S}' and Q_{E}' in Eq.(13).

IV. ONE-LAYER BAROCLINIC DIABATIC VORTICITY EQUATIONS

Subtracting the anomaly equations corresponding to Eq.(3), the signs of u' and v' on the upper troposphere are considered opposite to those on the lower following Eq.(16) and applicable expressions for low-level flow field. The integral mid-values of u_*' and v_*' in the flow field of disturbance are almost negligible as compared with u_* and v_*. As a result, the values of u_* and u_* are close to $\bar{u}_*$ and $\bar{v}_*$ so that u_*' and v_*' are ignored. Substituting Eq.(14) into the corresponding equation for anomaly in Eq.(3) and denoting $\phi = \Phi - \Phi_m$ in which Φ_m is the climatological mean of Φ, we have

$$\begin{aligned}& \frac{\partial \phi_{\mathrm{T}}'}{\partial t} + u_*\frac{\partial \phi_{\mathrm{T}}'}{\partial x} + v_*\frac{\partial \phi_{\mathrm{T}}'}{\partial y} + c_{\mathrm{A}}^2\left(\frac{\partial u_{\mathrm{T}}'}{\partial x} + \frac{\partial v_{\mathrm{T}}'}{\partial y}\right) \\ & = -(1+\delta)\frac{4\eta_0 a\sigma T_*^3}{R}\phi_{\mathrm{T}}' - \eta_0\frac{c_d c_p A\rho_0 U_0}{R}\phi_{\mathrm{T}}' - [1-(1-\delta)\eta_0\eta] \\ & \cdot\frac{0.4\eta_4 g L b_1 (q_{s\mathrm{B}} - q_{s\mathrm{T}})\eta_0}{4H}\zeta_{\mathrm{T}}' + \delta[4\eta_0\sigma A T_{\mathrm{S}}' + c_d c_p A\rho_0 U_0]T_{\mathrm{S}}'\end{aligned} \tag{14}$$

$$c_{\mathrm{A}}^2 = c_a^2 - \varepsilon\frac{R}{c_p}\frac{q_{s\mathrm{B}} - q_{s\mathrm{T}}}{4}(1-\eta_0\eta)L\eta_4\frac{p_N - p_{\mathrm{T}}}{p_{\mathrm{N}}} \tag{15}$$

Here, Eq.(14) is the so-called one-layer clinic, diabatic "shallow water" equation, which forms an enclosed set with the 250 hPa motion equation (for low-latitudes) or the quasi-geostrophic vorticity equation. With the underlying land condition and exclusion of a particular heating source

$\tilde{Q}'_L$, the latter three terms in Eq.(14) are negligible. For Eq.(15), the appearance of condensation makes $c_i < c_a$ and c_i =38 m/s when taking $(q_{sB} - q_{sT})$ as 30×10^{-3} and ε=0.5. c_i is often considered the theoretic value for the Kelvin wave velocity, which is in effect close to the observed value for the tropopause and lower stratosphere. The stream function of disturbance ψ for p_T =250 hPa is defined in

$$u'_T = -\partial\psi / \partial y; \quad v'_T = -\partial\psi / \partial x \tag{16}$$

$\nabla^2\psi$ is the anomalous vorticity. Subtracting the 250 hPa anomalous equation corresponding to Eq.(6) and removing the divergence term by combining Eq.(14), the quasi-geostrophic approximation is assumed: $\phi'_T = f_0\psi$ while $\left(\frac{\partial f_0}{\partial y}\right) = 0$ and $f + \bar{\zeta}_T$ is replaced by f. Then an equation about f is obtained as in

$$\left(\nabla^2 - \frac{f_0^2}{c_A^2}\right)\frac{\partial\psi}{\partial t} + \left(\bar{u}_T\nabla^2 + \beta + \frac{\partial\bar{\zeta}_T}{\partial y} - \frac{f_0^2}{c_A^2}u_*\right)\frac{\partial\psi}{\partial x} + \left(\bar{v}_T\nabla^2 - \frac{\partial\bar{\zeta}_T}{\partial x} - \frac{f_0^2}{\partial x} - \frac{f_0^2}{c_A^2}v_*\right)\frac{\partial\psi}{\partial y}$$
$$= \frac{\eta_0 f_0^2}{c_A^2}(1+\delta)\left[\frac{4a\sigma T_*^3}{R} + \frac{c_d c_p \rho_0 U_0}{R}\right]\psi + [1 - \eta_0\eta(1+\delta)]\frac{0.4g\eta_0 b_1}{8H}\cdot\eta_4 q_b L\nabla^2\psi$$
$$-\mu\nabla^4\psi - \delta_A\frac{f_0}{c_A^2}[4\eta_0\delta T_S^3] + \eta_0 c_d c_p \rho_0 U_0 T'_S \tag{17}$$

For this derived expression Eq.(17), the approximation discussed in the text above is used : $q_{sB}/2$ is substituted by $(q_{sB} - q_{sT})$.

Eq.(16) is just a one-layer baroclinic, diabatic vorticity equation. Like in Eq.(14), the terms T'_S and Q'_E can be ignored when the underlying surface is land; the last term $\bar{Q}'_L$ can also be dropped if there is not any particular need. Eq.(17) takes much resemblance to Charney's barotropic vorticity equation (Charney, 1974 & 1979). It is, however, a "duplicate" of the atmospheric baroclinity, as suggested by Charney. Eq.(17) is the "original", because it is deduced directly from the very nature of baroclinic atmosphere. It appears to be justifiable in employing 250 hPa rather than 500 hPa in describing the state of the atmosphere, as it is the case in Eq.(17) and Hoskins et al.

Assuming that Eq.(17) can be Fourier-expanded in a two-dimensional space:

$$\psi = \sum_{n,m=-\infty}^{\infty}\sum \psi_{m,n} e^{i(kx+ly)}, \quad k = \frac{2\pi n}{L}, \quad l = \frac{\pi m}{D} \tag{18}$$

where L is the length of dimensional circle and D is the width of wave. From Eq.(18), it is natural that

$$\nabla^2\psi = -\bar{K}^2\psi = -(\bar{k}^2 + \bar{l}^2)\psi \tag{19}$$

where $\tilde{k}^2$ and $\tilde{l}^2$ are the spectral mean for k^2 and l^2, respectively. By $\tilde{K}^2 = \tilde{k}^2 + \tilde{l}^2$, the spectral mean is defined by

$$(\widetilde{\bullet}) = \frac{\sum\sum_{n,m=-\infty}^{\infty} (\bullet)\psi_{m,n} e^{i(kx+ly)}}{\sum\sum_{n,m=-\infty}^{\infty} \psi_{m,n} e^{i(kx+ly)}} \tag{20}$$

By substituting Eq.(19) into Eq.(17) and replacing $\widetilde{K}^2$ with the ∇^2 operator, Eq.(17) is changed to a partial differential equation of the 1st order. It is thus easy to obtain its characteristic equation set by

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\overline{K}^2 c_A^2 \bar{u}_T + f_0^2 u_*}{\overline{K}^2 c_A^2 + f_0^2} - \frac{c_A^2}{\overline{K}^2 c_A^2 + f_0^2}\left(\beta + \frac{\partial \bar{\zeta}_T}{\partial y}\right) \tag{21}$$

$$\dot{y} = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\overline{K}^2 c_A^2 \bar{v}_T + f_0^2 v_*}{\overline{K}^2 c_A^2 + f_0^2} + \frac{c_A^2}{\overline{K}^2 c_A^2 + f_0^2}\frac{\partial \bar{\zeta}_T}{\partial x} \tag{22}$$

$$\dot{\psi} = -\frac{A}{\overline{K}^2 c_A^2 + f_0^2}\left[\frac{\eta_0 f_0^2}{c_A^2}\frac{4a\sigma T_*^3}{R}(1+\delta) + \frac{\delta c_d c_p \rho_0 U_0}{R} + \mu \widetilde{K}^4\right]\psi + \frac{\eta_0 f_0^2 / c_A^2}{\overline{K}^2 c_A^2 + f_0^2}$$

$$\cdot [1 - \eta_0 \eta (1-\delta)] \frac{0.4 g \eta_0 b_1}{8H} \eta_4 q_b L \widetilde{K}^2 \psi + \frac{\eta_0 f_0^2 \delta A}{c_A^2 (\overline{K}^2 c_A^2 + f_0^2)}$$

$$\cdot (4\sigma T_S^3 + c_b c_p \rho_0 U_0) T_S' \tag{23}$$

Meanwhile, the following form should also satisfy Eq.(17), or

$$\psi = \sum_{n=\pm|n|} \sum_{m=\pm|m|} \psi_{m,n} e^{i(kx+ly)} \tag{24}$$

With the substitution of Eq.(24) with Eq.(17), every expression in Eqs.(19)-(23) is valid so long as $\widetilde{K}^2$ is taken as $\widetilde{K}^2 = K^2 = \widetilde{k}^2 + \widetilde{l}^2$. In other words, Eq.(18) is composed of countable infinite expressions of Eq.(24), including $n=m=0$.

For three-dimensional systems in Eqs.(21), (22) and (23), ψ not being explicitly contained, Eqs.(22) and (23) act as relatively independent systems with two dimensions. They can be proved as the spectral mean phase velocity in the Rossby wave.

V. CENTER OF ACTION AND LOW FREQUENCY OSCILLATION

For stationary waves of $\dot{x} = \dot{y} = 0$, we have $\widetilde{K}^2 = K_S{}^2$, in which $\widetilde{K}^2$ is taken in context of Eq.(24). From Eqs.(21) and (22), we have

$$K_S^2 c_A^2 \bar{u}_T + f_0^2 u_* - c_A^2\left(\beta + \frac{\partial \bar{\zeta}_T}{\partial y}\right) = 0 \tag{25}$$

$$K_S^2 c_A^2 \bar{v}_T + f_0^2 u_* - c_A^2 \frac{\partial \bar{\zeta}_T}{\partial x} = 0 \tag{26}$$

Using Eqs.(25) and (26), Eqs.(21) and (22) are rewritten as

$$\frac{dx}{dt} = \frac{(\widetilde{K}^2 - K_S^2) c_A^2 \bar{u}_T}{\widetilde{K}^2 c_A^2 + f_0^2} \tag{27}$$

$$\frac{dy}{dt} = \frac{(\widetilde{K}^2 - K_S^2) c_A^2 \bar{v}_T}{\widetilde{K}^2 c_A^2 + f_0^2} \tag{28}$$

$\bar{u}_T$ and $\bar{v}_T$ are denoted by climatically mean stream function:

$$\bar{u}_T = -\frac{\partial \psi_S}{\partial y}, \quad \bar{v}_T = \frac{\partial \psi_S}{\partial x} \tag{29}$$

By setting

$$d\tau = \left(1 - \frac{K_S^2}{\widetilde{K}^2}\right) \frac{\widetilde{K}^2 c_A^2}{\widetilde{K}^2 c_A^2 + f_0^2} dt = r dt \tag{30}$$

Eqs.(27) and (28) are further written as

$$\frac{dx}{d\tau} = -\frac{\partial \psi_S}{\partial y} \tag{31}$$

$$\frac{dx}{d\tau} = \frac{\partial \psi_S}{\partial x} \tag{32}$$

The two-dimensional system [Eqs.(31) and (32)] is a Harmilton's system that is balanced at $\bar{u}_T = \bar{v}_T = 0$, or at $\frac{\partial \psi_S}{\partial x} = \frac{\partial \psi_S}{\partial y} = 0$ .i.e. at the center of action. For the elementary integral in Eqs.(31) and (32), ψ_S = const., each enclosed contour of ψ_S has its own periodic solution that corresponds to the variation of phase. Starting from its linear equation of approximation, the period around the balanced point is derived as in

$$P = \frac{2\pi}{\left(r \cdot \sqrt{\frac{\partial^2 \psi_S}{\partial x^2} \frac{\partial^2 \psi_S}{\partial y^2}}\right)_0} = \frac{2\pi}{\left(r \cdot \sqrt{-\frac{\partial^2 \bar{v}_T}{\partial x} \frac{\partial^2 \bar{u}_T}{\partial y}}\right)_0} \tag{33}$$

For the enclosure centers of $\frac{\partial^2 \psi_S}{\partial x^2}$ and $\frac{\partial^2 \psi_S}{\partial y^2}$, the signs are identical. By setting $\left|\frac{\partial \bar{v}_1}{\partial x}\frac{\partial \bar{u}_1}{\partial y}\right| \approx 10^{-11}\mathrm{s}^{-2}$, $\widetilde{K}^2 \approx 4 \times 10^{-12}\mathrm{m}^{-2}$, $K_S^2 \approx 2 \times 10^{-12}\mathrm{m}^{-2}$, $f_0^2 = 0\text{–}1 \times 10^{-4}\mathrm{s}^{-1}$, we have r=(1/6) – (1/2), and P=15 – 115 days. It is obvious that the center of action is the source that sends off the signal of low frequency oscillation.

In the current text, $-\widetilde{K}^2\psi$ is the vorticity disturbance and $f\psi$ the potential disturbance. The individual change in ψ reflects the disturbance in vorticity and potential, i.e. the variation of amplitude of wave disturbance. If T_S' is ignored and the amplitude is set to vary in proportion to $e^{\sigma_1 t}$, then $\sigma_1 = \dot{\psi}/\psi$. It is then clear that the development of low frequency oscillation depends on $\dot{\psi}$, which is, inferring from Eq.(23), made up of three terms, the first being long-wave radiation and frictional dissipation, the second being positive feedback by CISK condensation and the last being triggered-off SSTA. It is obvious that condensation latent heat over land provides the most important energy for the low frequency oscillation while the principle supplier over the sea shifts to latent heat release associated with positive SSTA and precipitation.

For a given point of location, there is alternative appearance of maximum positive and negative anomalies in a quasi-periodic cycle. As a result, there are opposite phases on two sides of the enclosed center while they are identical or similar to each other on the common boundary of two stationary, closed flow fields, due to continuity.

Ⅵ. HORIZONTAL PROPAGATION OF LOW FREQUENCY OSCILLATION

By defining Eq.(20), it is deduced that $\frac{\partial \widetilde{k}^2}{\partial k} = 2\widetilde{k}$, and from Eqs.(21) and (22), the group velocity c_g is derived in

$$c_{gx} = \dot{x} + \widetilde{k}\frac{\partial \dot{x}}{\partial k} = \frac{2\widetilde{K}^2 c_A^2 \bar{u}_T}{\widetilde{K}^2 c_A^2 + f_0^2} + \frac{\bar{l}^2 - \bar{k}^2}{\widetilde{K}^2 c_A^2 + f_0^2} c_A^2 \dot{x} \tag{34}$$

$$c_{gy} = \dot{x} + \widetilde{l}\frac{\partial \dot{y}}{\partial l} = \frac{2\widetilde{l}^2 c_A^2 \bar{v}_T}{\widetilde{K}^2 c_A^2 + f_0^2} - \frac{\bar{l}^2 - \bar{k}^2}{\widetilde{K}^2 c_A^2 + f_0^2} c_A^2 \dot{y} \tag{35}$$

or they can be simplified as $\vec{c}_g = \vec{c}_{g1} + \vec{c}_{g2}$, in which $\vec{c}_{g1}$ is the 1st term in Eqs.(33) and (34), and $\vec{c}_{g2}$ the 2nd term. As $\vec{c}_{g2}$ is a vector that is perpendicular to the phase velocity C, it directs to the right of C by 90°. When the phase velocity moves along the enclosed contour, $\vec{c}_{g2}$ converges towards the center or dissipates in all directions, playing no part in oriented transfer of low frequency energy. It is $\vec{c}_{g1}$ that correlates remotely and is called the oriented component of the group velocity, i.e.

$$c_{gx1} = \frac{2\tilde{k}^2 c_A^2}{\tilde{K}^2 c_A^2 + f_0^2} \bar{u}_T \tag{36}$$

$$c_{gy1} = \frac{2\tilde{l}^2 c_A^2}{\tilde{K}^2 c_A^2 + f_0^2} \bar{v}_T \tag{37}$$

Setting the angle as α included between the horizontal rays of propagation of low frequency oscillation and zonal circle and that as α_0 included between the mean climatological stream lines and zonal circle, we have by inferring Eq.(37) that

$$\tan\alpha = \frac{c_{gy1}}{c_{gx1}} = \frac{\tilde{l}^2}{\tilde{k}^2}\frac{\bar{v}_T}{\bar{u}_T} = \left[\frac{\tilde{l}^2}{\tilde{k}^2}\right]\tan\alpha_0 \tag{38}$$

Generally, as $\tilde{l}^2 \geqslant \tilde{k}^2$, we have $\alpha \geqslant \alpha_0$. In the meantime, $\alpha=0$ when $\alpha_0=0$. The rays turn in direction at the same longitude as troughs and ridges. From Eqs.(36), (37), or (38), it is known that the rays follow such a route that they start from the oscillatory source and cross northward across the streamline before turning for the south at the top of the ridge or north of it (for the southerly); they cut southward across the streamline and turn for the north at the bottom of trough or south of it.

VII. COMPARISON WITH DIAGNOSTIC RESULTS

The theory presented above seems more concrete and reasonable than that of great circle route in interpreting low frequency travelling track with observed data. For example, the pattern of PNA moves first to the north and then towards south across the ridge of Alaska, where it travels in the northerly east of the ridge and cuts across the bottom of North American Trough (the jet area). In the pattern of EU, however, the rays first move over the Siberia to the southeast until the jet area of East Asian Trough (northern branch). Part of the ray looks like a great arc but the whole track is not always in the shape of a great circle. Take the teleconnection (dipole) pattern of stationary waves for example. Each of the highs on the zone of the subtropical high is a closed center itself. When applied with the theory in this work, the low frequency variations take on a mode of opposite phase on the flanks of the zone but a mode of identical phase in the intermediate area between two high cells (due to the existence of continuity). It is then natural that there are centers of anomalous correlation in alternation of positive and negative values (+ – + –) along the zone of the subtropical high and the low frequency variations are just the opposite in phase on the south and north sides of it.

The preceding inference agrees well with the diagnostic results by Hsu and Lin (1992) who work on the teleconnection of low frequency variation at 250 hPa in winter.

VIII. CONCLUDING REMARKS

On basis of one-layer baroclinic diabatic equation of vorticity, it is proved that a closed center of climatological mean flow field is the source that generates signals of low frequency oscillation, whose energy is supplied by condensation latent heat associated with precipitation and sensible heat transfer by sea surface temperature in addition to the opposite-phased structure on both sides of the enclosed center. Deriving the formulae for routes of horizontal propagation of low frequency oscillation [See Eqs.(36) – (38)), it is suggested that the wave ray emitting from the source of low frequency oscillation are moving northward (in the case of southerly) across the streamlines before making recurvature at the top of the ridge (or the north of it); it is moving southward (in the case of northerly) across the streamlines and heading towards the northeast at the bottom of the trough. The finding is generally consistent with diagnostic results and thus more concrete and reasonable than the great circle theory would be.

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