ON THE COMPUTATION OF THE VERTICAL VELOCITY AT THE TOP OF PBL IN LOW LATITUDE AREAS

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ABSTRACT

This paper suggests a method to compute the vertical velocity at the top of PBL in low latitude areas by use of the wind data on (sea) surface and some characteristics of the vertical velocity are shown. The results show the important roles played by the inertial forces and β effect.

Key words: vertical velocity on top of PBL, low latitude, sea surface wind

I. INTRODUCTION

The vertical velocity at the top of the planetary boundary layer of the atmosphere is an important parameter that determines the exchange among various physical quantities between the PBL and the free atmosphere. Research in this respect has been mainly decided by the geostrophic wind field that is based on the boundary layer model being free of the effects of inertial force, whether the Charney-Eliassen (C-E) formula, derived from the classic Ekman solution, or the similarity theory for the boundary layer (Zhao, 1994), is used. The methods are generally applicable to mid-latitudes. In low latitudes, it is more difficult to determine the geostrophic winds due to smaller geostrophic parameters and pressure gradient. The use of the methods prove to be ineffective for motions especially in low latitudes where the inertial force is comparable with other forces. Additionally, the β effect is obvious there that it is necessary to study its effects on vertical velocity at the top of boundary layer. In contrast to mid-latitudes, the work seems more significant for low latitudes where the cumulus convection plays a vital role in the synoptic development there. For the current treatment, the cumulus convection is derived through the computation of the amount of water vapour flowing into the free atmosphere from the boundary layer on basis of the pumping effects at the top of the boundary layer. In the meteorology for low latitudes, the importance of computing the velocity is not emphasized because of the persistent, wide use of the C-E formula in the study of low-latitude convection by cumulus and of some other problems in the field of low-latitude dynamics. It is attempted in this paper that the surface rather than geostrophic winds are used to compute the vertical velocity at the top of the layer, taking into account in the meantime the inertial and β effects. A number of relevant characteristics are obtained. The surface winds here follow the conventional value at the height of 10 m and especially denote that over the sea since the low-latitudes is dominated by the ocean.

II. DERIVATION OF FORMULA

Generally, the vertical velocity at the top of the boundary layer is derived in the con-

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tinuous equations using the solutions of the motion equation for PBL, like the Charney-Eliassen Formula. A negative aspect involved in this method is that the result is much dependent upon the form of coefficients of turbulent exchange. The derivation here is done directly by the surface stress from the vertical integration rather than the solutions, concerning the PBL motion equations. Neglecting the inertial force, the equilibrium equations involving the pressure gradient, frictional, and Coriolis forces are shown in

$$\frac{d\tau_x}{dz} + f(v - v_g) = 0 \tag{1}$$

$$\frac{d\tau_y}{dz} - f(u - u_g) = 0 \tag{2}$$

where τ_x, τ_y are the turbulent shearing stress in x, y directions, u_g, v_g are the geostrophic components with the x axis pointing to the east and y axis the north. Eq. (2) is differentiated with respect to x with subtraction of differentiation with respect to y before integration over the height from the ground surface to the top of the boundary layer. Considering that $w = 0$ at the surface and the stress equals to zero at the top, the vertical velocity there (in the continuous equations) then becomes

$$\vec{w}_h = \frac{1}{f}\nabla \times \vec{\tau}_0 + \frac{\beta\tau_{x0}}{f^2}\vec{k} \tag{3}$$

or

$$w_h = \frac{1}{f}\left(\frac{\partial\tau_{y0}}{\partial x} - \frac{\partial\tau_{x0}}{\partial y}\right) + \frac{\beta\tau_{x0}}{f^2}, \tag{4}$$

the subscript "0" denotes the value at the surface. Though the incompressible equation of continuation is used in the derivation of Eq. (3), the shape of the wind profile within the boundary layer is needless to know, i.e. relevant model is not required. If the β effects are negligible, Eq. (4) becomes the form widely cited in documentation. The surface shearing stress τ_{x0}, τ_{y0} can be expressed by the ground (and sea) surface winds. The formula seeking w_h by winds at ground (and sea) surface is derived by use of Eq. (4). Generally, the inertial force is introduced to Eqs. (1) and (2) so that

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{d\tau_x}{dz} - f(v - v_g) = 0 \tag{5}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - \frac{d\tau_y}{dz} + f(u - u_g) = 0 \tag{6}$$

where vertical advection is omitted for approximation. First of all, the term of inertial force is linearized. According to the PBL theory (Зилитинкевич, 1970), the angle of intersection is usually small between the surface winds and upper limit of PBL for low-latitude sea surface; observations show that there is a mixed layer in the PBL there in which the potential temperature varies over a mild range (Krishnamurti, 1987); strong turbulent mixing restrains the variation of winds with height in PBL above surface layer and to a more remarkable extent over body of warm water (Bond, 1992). In this work, the vertical mean of $\bar{u}$、$\bar{v}$ of the horizontal velocity of winds at the height range of PBL are used to replace the advection velocity in Eqs. (5) and (6) for linearization. Then, the precedure deriving the vorticity equation is applied to Eqs. (5) and (6) before the vorticity equations, now linearized and containing the term of stress, are integrated from $z=0$ to $z=h$ (PBL top). Applying the equation of continuity, we obtain the vertical velocity at the top of PBL in

$$w_h=\frac{\frac{\partial\tau_{y_0}}{\partial x}-\frac{\partial\tau_{x_0}}{\partial y}}{f+\bar{\zeta}}+\frac{h\frac{d\bar{\zeta}}{dt}}{f+\bar{\zeta}}+\frac{\beta h}{f(f+\bar{\zeta})}\frac{d\bar{u}}{dt}+\frac{\beta\tau_{x_0}}{f(f+\bar{\zeta})} \tag{7}$$

where $\bar{\zeta}=\frac{1}{h}\int_0^h\zeta dz$ is the mean vertical vorticity of PBL as in

$$\frac{d\bar{\zeta}}{dt}\equiv\frac{\partial\bar{\zeta}}{\partial t}+\bar{u}\frac{\partial\bar{\zeta}}{\partial x}+\bar{v}\frac{\partial\bar{\zeta}}{\partial y}\qquad\frac{d\bar{u}}{dt}\equiv\frac{\partial\bar{u}}{\partial t}+\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}.$$

In Eq(7), the first and fourth terms correspond to Eq. (4) that is free of the inertial force but with the denominator changed to $f+\bar{\zeta}$. For areas outside the low latitudes, $\bar{\zeta}$ is much smaller than f in the denominator of Eq. (7) so that the sum of the first and fourth terms corresponds to Eq. (4). Apparently, the introduction of $\bar{\zeta}$ in the terms accounts for the inertial force. The identical order of magnitude for f and $\bar{\zeta}$ shown for low latitudes suggests the same thing between the inertial force and the frictional term. Apart from it, the inertial force is also present in the second and third terms of Eq. (7).

Treating τ_{x0}、τ_{y0} in Eqs. (7) and (4) with simple parameterization yields

$$\vec{\tau}_0=c_DV\vec{V} \tag{8}$$

where C_D is the drag coefficient, $V=\sqrt{u^2+v^2}$ is the total wind velocity, and V is the wind velocity at the ground (and sea) surface. Then,

$$\tau_{x0}=c_DuV\qquad\tau_{y0}=c_DvV, \tag{9}$$

C_D can be simplified as constant or, with more complicated situation, set as the function of stratification and wind velocity. As stratification parameter is not readily available at the near-surface layer, C_D is set at constant, of 1.3×10^{-3}, to be exactly, according to the latest data (Hedde, 1994). Substituting Eq. (9) into Eq. (7) or (4) leads to solution of the first and fourth terms in Eq. (7) using the distribution of ground (and sea) surface winds while the second and third terms, inaccurately accessed, are estimated based on the same distribution. Since the vertical mean winds are, as illustrated above, already linearized in the PBL at low latitudes, the second and third terms in Eq. (7) can be estimated by neglecting the angle of intersection between the vertical mean winds and ground (or sea) surface winds and approximating the vertical mean winds of the PBL. The wind velocity at the sea surface is generally about 70 % of that at the upper limit of the PBL (Zhao, 1987). The surface wind is estimated to be around 85 % of the mean wind in PBL. It is, therefore, possible to estimate the vertical mean winds of the PBL given the direction and velocity at the surface of ground (and sea) and further on to determine the second and third terms and $\bar{\zeta}$, with values being inaccurate but right in the order of magnitude.

Ⅲ. COMPUTATIONAL EXAMPLE AND RESULTS WITHOUT ACCOUNT OF INERTIAL FORCE

As an example of computation, the first step is to set in a constant easterly flow, or, $v=0$, in which the wind velocity acts only as the function of y. Thus, the inertial force is made zero so that Eq. (4) is applicable. From Eqs. (4) and (9), we get

$$w_h = \frac{c_D}{f}\left(\frac{\partial vV}{\partial x} - \frac{\partial uV}{\partial y}\right) + \frac{\beta c_D u V}{f^2}. \tag{10}$$

As a particular case, w_h at 10°N is computed with the assumption of $u=-8$ m/s, $\frac{\partial u}{\partial y} = \pm 10^{-5}\mathrm{s}^{-1}$, and as $f=2.5\times10^{-5}s^{-1}$, $\beta=2.2\times10^{-11}\mathrm{s}^{-1}\mathrm{m}^{-1}$, the result may be derived from Eq. (10) as in

$$w_h = \pm\frac{2c_D}{f}u\frac{\partial u}{\partial y} - \frac{\beta c_D u^2}{f^2} = \begin{cases} -1.12\times10^{-2}\mathrm{m/s} \\ 5.4\times10^{-3}\mathrm{m/s} \end{cases}.$$

Specifically, when $\frac{\partial u}{\partial y}>0$, or, the easterly decreases towards the north to display anticyclonic vortex, $w_h<0$; $w_h>0$ otherwise. Of the contribution, the β term always enables $w_h<0$ while the westerly makes $w_h>0$ hold without exception.

With the same vorticity, the small measure of f at the low latitudes always increases the absolute value of w_h, which decreases accordingly as the wind velocity becomes small. For the particular case shown above, the first term is about 2 to 3 times larger than the second one in Eq. (10). It is clear, therefore, that the β effects cannot be neglected in low latitudes.

Ⅳ. COMPUTATIONAL EXAMPLE AND RESULTS WITH INERTIAL FORCE

Let $u=-8$ m/s, $\frac{\partial u}{\partial x}=0$, $v=1.6$ m/s be constant, $\frac{\partial u}{\partial y}=\pm10^{-5}\,\mathrm{s}^{-1}$ and remains valid so that there is the inertial force. The vertical mean winds are estimated with the method presented in Section Ⅱ, that is, $\bar{u}=-9.4$ m/s, $\bar{v}=1.8$ m/s. Let $\frac{\partial \bar{u}}{\partial y} = \pm1.2\times10^{-5}\mathrm{s}^{-1}$, $\bar{\zeta}=\mp1.2\times10^{-5}s^{-1}$ is obtained. The computation is still for 10°N. As $\frac{d\bar{\zeta}}{dt}=0$, and the second term in Eq. (7) equals to zero, the first term on the right hand side of Eq. (7) is derived from Eq. (9) and set $h=10^3$ m and we have that

$$w_h = \sum_{i=1}^{4} w_i = \begin{cases} -0.016+0+1.42\times10^{-3}-5.2\times10^{-3}=-0.02\ \mathrm{m/s} \\ 5.61\times10^{-3}+0-5\times10^{-4}-1.82\times10^{-3}=3.29\times10^{-3}\ \mathrm{m/s} \end{cases}.$$

It is known, therefore, that the first and fourth terms are more important in this case and as the case before, the sign of w_h is determined by the area of positive and negative vorticity. The β effects are not negligible for this case.

Let's examine another case. Let $u=-8$m/s, $v=-6$m/s, $\frac{\partial u}{\partial y}=\pm10^{-5}\mathrm{s}^{-1}$, $\frac{\partial^2 u}{\partial y^2}=10^{-11}\mathrm{s}^{-1}\mathrm{m}^{-1}$, then the second and third terms in Eq. (7) become nonzero. Repeating the forthgoing treatmeat, we have that

$$w_k = \begin{cases} -16\times10^{-3}+6.46\times10^{-3}+5.73\times10^{-3}-5.74\times10^{-3}=-9.55\times10^{-3}\,\mathrm{m/s} \\ 5.62\times10^{-3}+2.27\times10^{-3}-2.01\times10^{-3}-2.02\times10^{-3}=3.87\times10^{-3}\,\mathrm{m/s} \end{cases}.$$

It is also obvious that the magnitude of the second and third terms is too large to be neglected.

V. COMPUTATIONAL EXAMPLE OF EASTERLY

Assuming that the air current is expressed by

$$\begin{cases} u = U(y) + A\cos(kx+\omega t) \\ v = -B\sin(kx+\omega t) \end{cases} \tag{11}$$

where $U(y)$ is the zonal basic flow and set $y=0$ at the point of computation (10°N). Let $U(0)=-7\,\mathrm{m/s}$, $(\frac{dU}{dy})_0=\pm10^{-5}s^{-1}$, $k=2.1\times10^{-6}m^{-1}$, or, a wavelength of 3000 km, a cycle of 4 days, an assumptiom that corresponds to $\omega=1.82\times10^{-5}s^{-1}$, $A=B=1$ m/s. The preceding precedure is used to treat the vertical mean winds and the term of $\bar{\zeta}$ for derivation of the wave at $\sin(kx+\omega t)=1$, $\cos(kx+\omega t)=0$ by

$$w_k=\sum_{i=1}^{4}w_i=\begin{cases} -1.38\times10^{-2}+0.2\times10^{-3}-0.86\times10^{-3}-4.35\times10^{-3}=-1.8\times10^{-2}\mathrm{m/s} \\ 5\times10^{-3}+0.07\times10^{-3}+0.3\times10^{-3}-1.25\times10^{-3}=4.1\times10^{-3}\mathrm{m/s} \end{cases}$$

Due to a small amplitude of wave fluctuation the first and fourth terms are significant in this case, w_k is computed at $\begin{cases} -1.94\times10^{-2}\,\mathrm{m/s} \\ 4.5\times10^{-3}\,\mathrm{m/s} \end{cases}$ if $A=B=2$ m/s is taken, a result that is somewhat larger than the previous one, and the second and third terms in Eq. (7) are more than 3 times as large as that when $A=B=1$ m/s.

With $A=B=1$ m/s, the wave is computed at a point where $\sin(kx+\omega t)=0$, $\cos(kx+\omega t)=1$ and we get

$$w_k=\sum_{i=1}^{4}w_i=\begin{cases} -13.3\times10^{-3}+0+0-3.9\times10^{-3}=-1.72\times10^{-2}\mathrm{m/s} \\ 5\times10^{-3}+0+0-1.2\times10^{-3}=3.8\times10^{-3}\mathrm{m/s} \end{cases}$$

Though the second and third terms are zero in the case, they are different from Eq. (4), the case without the inertial force, by showing that

$$w_k=\begin{cases} -7.27\times10^{-3}\mathrm{m/s} \\ 5.25\times10^{-3}\mathrm{m/s} \end{cases}$$

It is obvious that the inertial force still plays an important role.

VI. CONCLUSIONS

The advantage in using ground (and sea) surface winds in the computation of vertical velocity at the top of PBL is that the geostrophic winds, which is difficult to obtain precision results due to not only the scarcity of sounding and pressure data over low-latitude oceans, but also the impossibility of exact determination of the pressure gradient it-

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