ON PROBLEMS OF HIGHLY TRUNCATED LOW-SPECTRAL MODEL IN STUDIES OF MULTIPLE EQUILIBRIA IN THE ATMOSPHERE

Lu Keli (吕克利) and Wang Boqiang (王柏强)

Department of Atmospheric Sciences, Nanjing University, Nanjing, 210008

Received 18 November 1994, accepted 17 March 1995

ABSTRACT

The multiple atmospheric equilibria are studied by using a barotropic vorticity equation with thermal forcing and dissipative effects. Different low-spectral models are used to discuss the variations of the equilibrium states, with the results that the multiple features of equilibrium states depend heavily on the truncations of the spectra, and the low-spectral model has obvious shortcomings in solving non-linear equations, suggesting that one has to be very careful to employ highly truncated low-spectral model in approximating partial differential equations.

Key words: low-spectral model, equilibrium state, partial differential equation

I. INTRODUCTION

Since the truncated spectral method was suggested by Lorenz (1960) the highly truncated low-spectral model has repeatedly been used to examine equilibrium states in non-linear system with forcing and dissipative effects. In 1979 there were papers (Vickroy and Dutton, Wiin-Nielsen, Charney and DeVore) in succession that dealt with the truncated spectral models to solve non-linear vorticity equation. Similar papers were also published in China (Miao and Ding, 1985). These papers studied the responses of atmosphere to the thermal forcing and the topography and discussed the sudden change of circulations. However, the solutions of ordinary differential equation derived by method of the finite truncated spectra are not necessarily the approximate solutions of the partial differential equation, and the increase of the truncated spectra may cause change in the equilibrium states. As a preliminary study we will examine numerical solution of the truncated spectral models with 3, 4, 5, and 6 spectra respectively, and discuss the variations of the equilibrium states.

II. BASIC EQUATIONS

The quasi-geostrophic vorticity equation with thermal forcing and dissipative effect may be written as follows

$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi, \nabla^2\psi) + \beta\frac{\partial\psi}{\partial x} + K\nabla^2\psi - K\nabla^2\psi^* = 0 \tag{1}$$

where $-K\nabla^2\psi^*$ represents the vorticity effect caused by thermal forcing, $K\nabla^2\psi$ is the Ekman-friction dissipation, $\beta = L\mathrm{ctg}\varphi_0/a$, L is the horizontal scale of atmospheric motions, a is the radius of the earth, $K = D_E/2H$, D_E is the thickness of the Ekman layer, H is the scale height of atmosphere.

Consider the motion in a rectangular region: $x\ (0, 2\pi)$, $y\ (0, \pi)$. Appropriate boundary conditions for such a channel are

$$\frac{\partial\psi}{\partial x}=0 \qquad y=0,\pi$$

ψ is periodic variation at $x=0$ and 2π.

The basic functions for the above boundary conditions may be taken to be orthonormal eigenfunctions F_i of Laplacian operator, which obey the following equation

$$\nabla^2 F_i+\lambda_i F_i=0. \tag{2}$$

Expanding ψ and ψ^* in terms of the basic functions F_i, namely, setting

$$\begin{pmatrix}\psi\\ \psi^*\end{pmatrix}=\sum_{i=1}^{\infty}\begin{pmatrix}\psi_i\\ \psi_i^*\end{pmatrix}F_i \tag{3}$$

and substituting (3) into (1) gives

$$\sum_{i=1}^{\infty}\lambda_i\frac{\partial\psi_i}{\partial t}F_i+\sum_{k=1}^{\infty}\psi_k\frac{\partial F_k}{\partial x}\sum_{j=1}^{\infty}\lambda_j\psi_j\frac{\partial F_j}{\partial y}-\sum_{k=1}^{\infty}\psi_k\frac{\partial F_k}{\partial y}\sum_{j=1}^{\infty}\lambda_j\psi_j\frac{\partial F_j}{\partial x}-\beta\sum_{i=1}^{\infty}\psi_i\frac{\partial F_i}{\partial x}$$
$$=K\sum_{i}^{\infty}\lambda_i F_i(\psi_i^*-\psi_i). \tag{4}$$

Multiplying the equation (4) by F_s, then integrating over the region D, and utilizing

$$\iint_D F_s F_r d\vec{x}=\begin{cases}0 & (s\neq r)\\ 1 & (s=r)\end{cases}$$

yields

$$\lambda_s\dot{\psi}_s+\sum_k\sum_m\lambda_m\psi_k\psi_m\iint_D\left(\frac{\partial F_k}{\partial x}\frac{\partial F_m}{\partial y}-\frac{\partial F_k}{\partial y}\frac{\partial F_m}{\partial x}\right)F_s d\vec{x}$$
$$-\beta\sum_k\psi_k\iint_D\frac{\partial F_k}{\partial x}F_s d\vec{x}=K\lambda_s\psi_s^*-K\lambda_s\psi_s. \tag{5}$$

Letting

$$D_{kms}=\iint_D\left(\frac{\partial F_k}{\partial x}\frac{\partial F_m}{\partial y}-\frac{\partial F_k}{\partial y}\frac{\partial F_m}{\partial x}\right)F_s d\vec{x}$$
$$=\frac{1}{2\pi^2}\int_0^{\pi}\int_0^{2\pi}\left(\frac{\partial F_k}{\partial x}\frac{\partial F_m}{\partial y}-\frac{\partial F_k}{\partial y}\frac{\partial F_m}{\partial x}\right)F_s dx\,dy$$

gives $D_{kms}=D_{msk}=D_{skm}=-D_{mks}$.

Setting

$$C_{ks}=\frac{1}{2\pi^2}\int_0^{\pi}\int_0^{2\pi}\frac{\partial F_k}{\partial x}F_s dx\,dy,$$

Eq. (5) becomes

$$\dot{\psi}_s=\lambda_s^{-1}\Big[\sum_k\sum_m(\lambda_k-\lambda_m)D_{kms}\psi_k\psi_m$$
$$+\beta\sum_k C_{ks}\psi_k+K\lambda_s(\psi_s^*-\psi_s)\Big]. \qquad k<m \tag{6}$$

Ⅲ. TRUNCATED SPECTRAL MODELS

1. *The spectral model with 6 basic functions*

Choose the orthonormal functions F_i in the following forms

$$F_1 = F_A = \sqrt{2}\cos y, \qquad F_2 = F_K = 2\cos nx \sin y$$
$$F_3 = F_L = 2\sin nx \sin y, \qquad F_4 = F_C = \sqrt{2}\cos 2y$$
$$F_5 = F_M = 2\cos nx \sin 2y, \qquad F_6 = F_N = 2\sin nx \sin 2y,$$

we obtain from (2)

$$\lambda_1 = 1, \qquad \lambda_2 = \lambda_3 = 1 + n^2, \qquad \lambda_4 = 4, \qquad \lambda_5 = \lambda_6 = 4 + n^2$$
$$D_{132} = 8\sqrt{2}\,n/3\pi, \qquad D_{354} = 64\sqrt{2}\,n/15\pi$$
$$D_{165} = 32n/15\pi, \qquad D_{246} = 64\sqrt{2}\,n/15\pi$$
$$C_{23} = -C_{65} = -C_{32} = C_{56} = -n$$

and for the others, $D_{lms} = 0, C_{ls} = 0$.

With these choices, Eq. (6) becomes

$$\begin{cases} \dot{\psi}_A = -K(\psi_A - \psi_A^*) \\ \dot{\psi}_K = -(a_1\psi_A - \beta_1)\psi_L - b_1\psi_C\psi_N - K(\psi_K - \psi_K^*) \\ \dot{\psi}_L = (a_1\psi_A - \beta_1)\psi_K + b_1\psi_C\psi_M - K(\psi_L - \psi_L^*) \\ \dot{\psi}_C = \varepsilon(\psi_K\psi_N - \psi_L\psi_M) - K(\psi_C - \psi_C^*) \\ \dot{\psi}_M = -(a_2\psi_A - \beta_2)\psi_N - b_2\psi_C\psi_L - K(\psi_M - \psi_M^*) \\ \dot{\psi}_N = (a_2\psi_A - \beta_2)\psi_M + b_2\psi_C\psi_K - K(\psi_N - \psi_N^*) \end{cases} \tag{7}$$

where

$$D_{AKL}/5 = D_{AMN}/4 = D_{CKN}/8 = D_{CML}/8 = 8\sqrt{2}\,n/15\pi$$
$$b_1 = n^2 D_{CKN}/(n^2+1), \qquad b_2 = (n^2-3)D_{CKN}/(n^2+4)$$
$$a_1 = n^2 D_{AKL}/(n^2+1), \qquad a_2 = (n^2+3)D_{AMN}/(n^2+4)$$
$$\varepsilon = 3D_{CKN}/4, \qquad \beta_1 = \beta n/(n^2+1), \qquad \beta_2 = \beta n/(n^2+4).$$

It is easily seen that the solutions ψ_i of (7) can be written as sum of the stationary solutions $\overline{\psi}_i$ and evolutionary solutions $\varphi_i(t)$

$$\psi_i = \overline{\psi}_i + \varphi_i(t) \tag{8}$$

Substituting (8) into (7) gives the equations with respect to $\overline{\psi}_i$:

$$\begin{cases} \overline{\psi}_A - \psi_A^* = 0 \\ d_1\overline{\psi}_L + b_1\overline{\psi}_C\overline{\psi}_N + K(\overline{\psi}_K - \psi_K^*) = 0 \\ d_1\overline{\psi}_K + b_1\overline{\psi}_C\overline{\psi}_M - K(\overline{\psi}_L - \psi_L^*) = 0 \\ \varepsilon(\overline{\psi}_K\overline{\psi}_N - \overline{\psi}_L\overline{\psi}_M) - K(\overline{\psi}_C - \psi_C^*) = 0 \\ d_2\overline{\psi}_M + b_2\overline{\psi}_C\overline{\psi}_K - K(\overline{\psi}_N - \psi_N^*) = 0 \\ d_2\overline{\psi}_N + b_2\overline{\psi}_C\overline{\psi}_L - K(\overline{\psi}_M - \overline{\psi}_M^*) = 0 \end{cases} \tag{9}$$

and the equations for φ_2 (neglecting the non-linear terms):

$$\begin{cases} \dot{\varphi}_A = -K\varphi_A \\ \dot{\varphi}_K = -d_1\varphi_L - a_1\overline{\psi}_L\varphi_A - b_1\overline{\psi}_C\varphi_N - b_1\overline{\psi}_N\varphi_C - K\varphi_K \\ \dot{\varphi}_L = d_1\varphi_K + a_1\overline{\psi}_K\varphi_A + b_1\overline{\psi}_M\varphi_C + b_1\overline{\psi}_C\varphi_M - K\varphi_L \\ \dot{\varphi}_C = \varepsilon(\overline{\psi}_K\varphi_N + \overline{\psi}_N\varphi_K - \overline{\psi}_M\varphi_L - \overline{\psi}_L\varphi_M) - K\varphi_C \\ \dot{\varphi}_M = -d_2\varphi_N - a_2\overline{\psi}_N\varphi_A - b_2\overline{\psi}_L\varphi_C - b_2\overline{\psi}_C\varphi_L - K\varphi_M \\ \dot{\varphi}_N = d_2\varphi_M + a_2\overline{\psi}_M\varphi_A + b_2\overline{\psi}_K\varphi_C + b_2\overline{\psi}_C\varphi_K - K\varphi_N \end{cases} \tag{10}$$

where $d_1 = a_1\overline{\psi}_A - \beta_1, d_2 = a_2\overline{\psi}_A - \beta_2$.

First consider the equations (9). For simplicity, setting $\psi_M^* = \psi_N^* = \psi_L^* = 0$, (9) becomes

$$\begin{cases} \overline{\psi}_A = \psi_A^* \\ d_1\overline{\psi}_L + b_1\overline{\psi}_C\overline{\psi}_N - K(\overline{\psi}_K - \psi_K^*) = 0 \\ d_1\overline{\psi}_K + b_1\overline{\psi}_C\overline{\psi}_M - K\overline{\psi}_L = 0 \\ \varepsilon(\overline{\psi}_K\overline{\psi}_N - \overline{\psi}_L\overline{\psi}_M) - K(\overline{\psi}_C - \psi_C^*) = 0 \\ d_2\overline{\psi}_N + b_2\overline{\psi}_C\overline{\psi}_L + K\overline{\psi}_M = 0 \\ d_2\overline{\psi}_M + b_2\overline{\psi}_C\overline{\psi}_K - K\overline{\psi}_N = 0. \end{cases} \tag{11}$$

It is easily seen that if $\overline{\psi}_C$ is assumed to be known we may obtain from (11) that

$$\begin{cases} \overline{\psi}_A = \psi_A^* \\ \overline{\psi}_K = [K^2b_1b_2\psi_K^*\,\overline{\psi}_C{}^2 + K^2(K^2 + d_2^2)\psi_K^*]/D \\ \overline{\psi}_L = [-Kd_2b_1b_2\psi_K^*\,\overline{\psi}_C{}^2 + Kd_1\psi_K^*(K^2 + d_2^2)]/D \\ \overline{\psi}_M = [-K^2b_2\psi_K^*(d_1 + d_2)\overline{\psi}_C]/D \\ \overline{\psi}_N = [Kb_1b_2{}^2\psi_K^*\,\overline{\psi}_C{}^3 + Kb_2\psi_K^*\overline{\psi}_C(K^2 - d_1d_2)]/D \end{cases} \tag{12}$$

where $D = b_1{}^2b_2{}^2\overline{\psi}_C{}^4 + 2b_1b_2(K^2 - d_1d_2)\overline{\psi}_C{}^2 + (K^2 + d_1{}^2)(K^2 + d_2{}^2)$.

Substituting (12) into the fourth equation of (11) gives

$$a_0\overline{\psi}_C{}^9 + a_1\overline{\psi}_C{}^8 + \cdots\cdots + a_8\overline{\psi}_C + a_9 = 0 \tag{13}$$

where

$$a_0 = b_1^4b_2^4, \quad a_1 = -b_1^4b_2^4\psi_C^*, \quad a_2 = 4b_1^3b_2^3(K^2 - d_1d_2), \quad a_3 = -4b_1^3b_2^3(K^2 - d_1d_2)\psi_C^*$$
$$a_4 = b_1^2b_2^2[4(K^2 - d_1d_2)^2 + 2(K^2 + d_1^2)(K^2 + d_2^2) - \varepsilon K^2b_2\psi_K^{*2}]$$
$$a_5 = -2b_1^2b_2^2\psi_C^*[2(K^2 - d_1d_2)^2 + (K^2 + d_1^2)(K^2 + d_2^2)]$$
$$a_6 = 2b_1b_2[2(K^2 - d_1d_2)(K^2 + d_1^2)(K^2 + d_2^2) - \varepsilon K^2b_2\psi_K^{*2}(K^2 - d_1d_2)]$$
$$a_7 = -4b_1b_2\psi_C^*(K^2 - d_1d_2)(K^2 + d_1^2)(K^2 + d_2^2)$$
$$a_8 = (K^2 + d_1^2)^2(K^2 + d_2^2)^2 - \varepsilon K^2b_2\psi_K^{*2}(K^2 + d_1^2)(K^2 + d_2^2)$$
$$a_9 = -(K^2 + d_1^2)^2(K^2 + d_2^2)^2\psi_C^*.$$

Solving (13) gives nine roots for $\overline{\psi}_C$. Substituting every real root of $\overline{\psi}_C$ into (12) yields an equilibrium state

$$\overline{\psi} = \overline{\psi}_A F_A + \overline{\psi}_K F_K + \overline{\psi}_L F_L + \overline{\psi}_C F_C + \overline{\psi}_M F_M + \overline{\psi}_N F_N. \tag{14}$$

For a given set of ψ_A^*, ψ_K^*, and ψ_C^*, (13) may have several real roots, it shows that there may exist multiple equilibrium states.

After the equilibrium states are obtained the stabilities of the equilibria are necessary to determine. According to the theory of the stability, the stability of equilibrium state depends on the characteristic equation of (10). When the roots of the characteristic equation have negative real part or the maximal real part of roots is less than zero the equilibrium states are stable. If one of the roots of characteristic equation has positive real part the equilibrium states are unstable. For this purpose introducing $\varphi_i(t) = R_i e^{\sigma t}(i = 1, 2, \cdots, 6)$ into (10) yields

$$\begin{cases} (\sigma + K)R_1 = 0 \\ (\sigma + K)R_2 + a_1\overline{\psi}_L R_1 + d_1 R_3 + b_1\overline{\psi}_N R_4 + b_1\overline{\psi}_C R_6 = 0 \\ -a_1\overline{\psi}_K R_1 - d_1 R_2 + (\sigma + K)R_3 - b_1\overline{\psi}_M R_4 - b_1\overline{\psi}_C R_5 = 0 \\ -\varepsilon\overline{\psi}_N R_2 + \varepsilon\overline{\psi}_M R_3 + (\sigma + K)R_4 + \varepsilon\overline{\psi}_L R_5 - \varepsilon\overline{\psi}_K R_6 = 0 \\ a_2\overline{\psi}_N R_1 + b_2\overline{\psi}_C R_3 + b_2\overline{\psi}_L R_4 + (\sigma + K)R_5 + d_2 R_6 = 0 \\ -a_2\overline{\psi}_M R_1 - b_2\overline{\psi}_L R_2 - b_2\overline{\psi}_K R_4 - d_2 R_5 + (\sigma + K)R_6 = 0. \end{cases} \tag{15}$$

Solving (15) gives the criterion of the stabilities of the equilibrium states.

2. *The spectral model with 5 basic functions*

For convenience of comparison, five basic functions F_A, F_K, F_L, F_C, F_N are taken from the above six basic functions so that the equations governing the stationary quantities for the case with five basic functions are easily obtained as follows

$$\begin{cases} \overline{\psi}_A = \psi_A^* \\ d_1\overline{\psi}_L + b_1\overline{\psi}_C\overline{\psi}_N + K(\overline{\psi}_K - \psi_K^*) = 0 \\ d_1\overline{\psi}_K - K\overline{\psi}_L = 0 \\ \varepsilon\overline{\psi}_K\overline{\psi}_N - K(\overline{\psi}_C - \psi_C^*) = 0 \\ b_2\overline{\psi}_C\overline{\psi}_K - K\overline{\psi}_N = 0. \end{cases} \tag{16}$$

The equations governing the evolutionary quantities are

$$\begin{cases} \dot{\varphi}_A = -K\varphi_A \\ \dot{\varphi}_K = -d_1\varphi_L - a_1\overline{\psi}_L\varphi_A - b_1\overline{\psi}_N\varphi_C - b_1\overline{\psi}_C\varphi_N - K\varphi_K \\ \dot{\varphi}_L = d_1\varphi_K + a_1\overline{\psi}_K\varphi_A - K\varphi_L \\ \dot{\varphi}_C = \varepsilon\overline{\psi}_N\varphi_K + \varepsilon\overline{\psi}_K\varphi_N - K\varphi_C \\ \dot{\varphi}_N = b_2\overline{\psi}_K\varphi_C + b_2\overline{\psi}_C\varphi_K - K\varphi_N. \end{cases} \tag{17}$$

As shown above when $\overline{\psi}_C$ is assumed to be known, we may obtain from (16)

$$\begin{cases} \overline{\psi}_K = K^2\psi_K^*/D \\ \overline{\psi}_L = Kd_1\psi_K^*/D \\ \overline{\psi}_N = Kb_2\overline{\psi}_C\psi_K^*/D \end{cases} \tag{18}$$

where $D = K^2 + d_1^2 + b_1 b_2 \overline{\psi}_C^2$.

Substituting (18) into the 4th equation of (16) yields

$$a_0 \overline{\psi}_C^5 + a_1 \overline{\psi}_C^4 + \cdots + a_4 \overline{\psi}_C + a_5 = 0 \tag{19}$$

where

$$\begin{aligned} a_0 &= b_1^2 b_2^2 \\ a_1 &= - b_1^2 b_2^2 \psi_C^* \\ a_2 &= 2(K^2 + d_1^2) b_1 b_2 \\ a_3 &= - 2(K^2 + d_1^2) b_1 b_2 \psi_C^* \\ a_4 &= (K^2 + d_1^2)^2 - \varepsilon K^2 b_2 \psi_K^{*2} \\ a_5 &= - (d_1^2 + K^2)^2 \psi_C^* . \end{aligned}$$

Solving (19) for $\overline{\psi}_C$ and substituting the $\overline{\psi}_C$ with real root into (18) gives an equilibrium state corresponding to the real root $\overline{\psi}_C$:

$$\overline{\psi} = \overline{\psi}_A F_A + \overline{\psi}_K F_K + \overline{\psi}_L F_L + \overline{\psi}_C F_C + \overline{\psi}_N F_N , \tag{20}$$

then the stability of the equilibria may be determined by the characteristic equation of (17).

3. *The spectral model with 4 basic functions*

Take F_A, F_K, F_C, and F_N as the basic functions, we obtain from (6)

$$\begin{cases} \overline{\psi}_A = \psi_A^* \\ b_1 \overline{\psi}_C \overline{\psi}_N + K(\overline{\psi}_K - \psi_K^*) = 0 \\ \varepsilon \overline{\psi}_K \overline{\psi}_N - K(\overline{\psi}_C - \psi_C^*) = 0 \\ b_2 \overline{\psi}_C \overline{\psi}_K - K \overline{\psi}_N = 0 \end{cases} \tag{21}$$

and the evolutionary quantities obey (dropping the nonlinear terms) the following equations

$$\begin{cases} \dot{\varphi}_A = - K \varphi_A \\ \dot{\varphi}_K = - b_1 \overline{\psi}_C \varphi_N - b_1 \overline{\psi}_N \varphi_C - K \varphi_K \\ \dot{\varphi}_C = \varepsilon \overline{\psi}_K \varphi_N + \varepsilon \overline{\psi}_N \varphi_K - K \varphi_C \\ \dot{\varphi}_N = b_2 \overline{\psi}_C \varphi_K + b_2 \overline{\psi}_K \varphi_C - K \varphi_N . \end{cases} \tag{22}$$

Assuming $\overline{\psi}_C$ in (21) to be known yields

$$\begin{cases} \overline{\psi}_K = K^2 \psi_K^* / D \\ \overline{\psi}_N = K b_2 \overline{\psi}_C \psi_K^* / D \end{cases} \tag{23}$$

where $D = K^2 + b_1 b_2 \overline{\psi}_C^2$, substituting (23) into (21) leads to

$$a_0 \overline{\psi}_C^5 + a_1 \overline{\psi}_C^4 + \cdots + a_4 \overline{\psi}_C + a_5 = 0 \tag{24}$$

where

$$a_0 = b_1^2 b_2^2$$
$$a_1 = -b_1^2 b_2^2 \psi_C^*$$
$$a_2 = 2b_1 b_2 K^2$$
$$a_3 = -2b_1 b_2 K^2 \psi_C^*$$
$$a_4 = K^4 - \varepsilon K^2 b_2 \psi_K^{*2}$$
$$a_5 = -K^4 \psi_C^* .$$

Analogously to the previous precedure, solving (24) gives an equilibrium state for a real root $\overline{\psi}_C$, i.e.,

$$\overline{\psi} = \overline{\psi}_A F_A + \overline{\psi}_K F_K + \overline{\psi}_C F_C + \overline{\psi}_N F_N, \tag{25}$$

and its stability may be determined by the characteristic equation of (22).

4. *The spectral model with 3 basic functions*

Taking F_A, F_K, F_C to be basic functions yields the equations governing the stationary variables

$$\begin{cases} \overline{\psi}_A = \psi_A^* \\ \overline{\psi}_K = \psi_K^* \\ \overline{\psi}_C = \psi_C^* \end{cases} \tag{26}$$

and the evolutionary variables obey the equations

$$\begin{cases} \dot{\varphi}_A = -K\varphi_A \\ \dot{\varphi}_K = -K\varphi_K \\ \dot{\varphi}_C = -K\varphi_C . \end{cases} \tag{27}$$

It is easily shown that for the present only one equilibrium state is obtained, namely,

$$\overline{\psi} = \psi_A^* F_A + \psi_K^* F_K + \psi_C^* F_C \equiv \psi^* . \tag{28}$$

Because the eigenvalue $\sigma = -K$ is negative the equilibrium state is stable.

Ⅳ. EXAMPLES OF CALCULATIONS

In order to compare the features of equilibria in the different truncated spectral models we calculated the equilibrium states for the four spectral models respectively. In calculations we take $K = 10^{-2}$, $L/a = 0.25$, $n = 2$; and let ψ_A^* and ψ_K^* be fixed values so that $\psi_A^* = 0.15$, $\psi_K^* = 0.40$, and ψ_C^* is taken to be changing from -0.3 to $+0.3$ for the spectral models with 6, 5, and 3 basic functions, but changing from -0.6 to $+0.6$ for the spectral models with 4 basic functions. Figs. 1, 2, and 3 give the varying curves of $\overline{\psi}_C$ with ψ_C^*. Fig. 1 shows the composite chart of equilibria in the plan $\overline{\psi}_C - \psi_C^*$ for the spectral model with 6 basic functions, and Fig. 2 shows those for the spectral models with 5 and 3 basic functions. The calculated results show that there are no difference between the equilibria of the truncated spectral models with 3 basic functions and 5 basic functions. The figures show that for the spectral model with 6 basic functions the responses of atmosphere may be 1, 2, 3, 4, and 5 equilibrium states when ψ_c^* varies, reflecting the multiplicity of the solution of nonlinear equation (1). For the continuous variation of ψ_c^* the responses of atmosphere have the feature of sudden change and it reflects quite well the catastrophe of the atmospheric circulations.

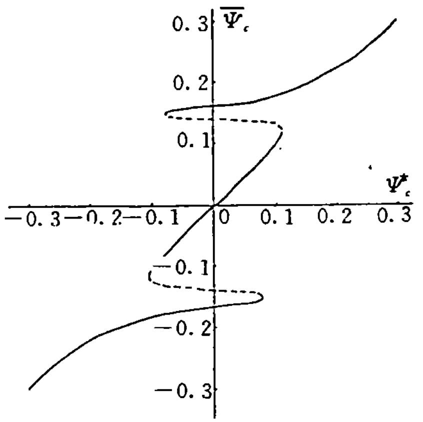

Fig. 1. Composite chart of equilibria in spectral models with 6 basic functions truncated.

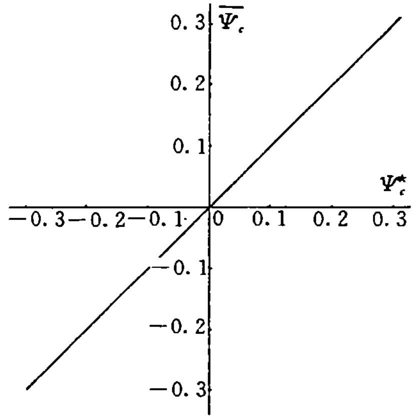

Fig. 2. The same as Fig. 1 except with 5 and 3 basic functions truncated.

However, Figs. 2 and 3 show that for the truncated spectral models with 5, 4, and 3 basic functions the responses of atmosphere do not possess the sudden change feature, and they are not corresponding with the real states of the atmospheric motions. Especially for the truncated spectral model with 4 basic functions the changes of stabilities of the equilibria take place, and unstable equilibria are more than the stable equilibria. It shows

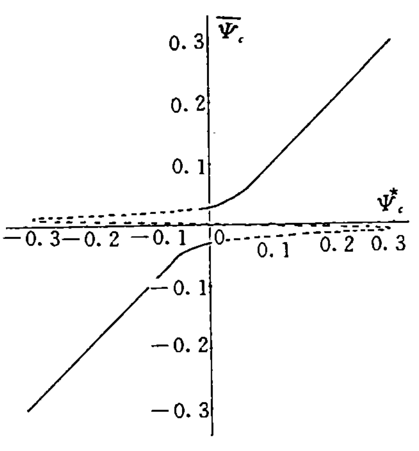

Fig. 3. The same as Fig. 1 except with 4 basic functions truncated.

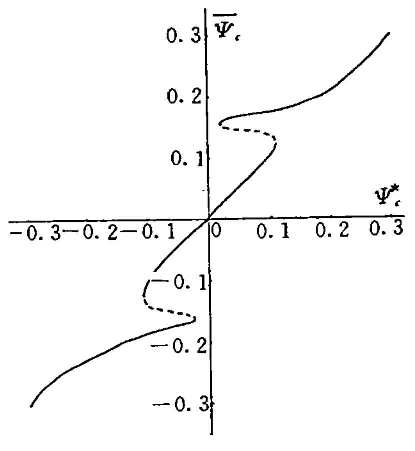

Fig. 4. The same as Fig. 1 except with 6 basic functions truncated. The solid lines are for steady equilibria and dashed lines for unsteady equilibria.

important effects of the truncated spectral model on the calculated results. In other words, for a determinate physical problem different results will be given if different truncated spectral models are used. It is obviously doubtful, and it may also be caused by using casually a set of finite ordinary differential equations (6, 5, 4, even 3 equations) to approximate the partial differential equations (1), because the solutions of these ordinary equations do not necessarily converge to the true solutions of the partial differential equation. Moreover, the approximate truncated spectral model may lose some principal components of the physical phenomena described by the original partial differential equation, and it may cause the distortion of the calculation results.

In order to further verify the calculated results above, we change the values of ψ_K^* and calculate again the above truncated spectral models. For the present study, take $\psi_K^* = 0.30$ and let ψ_c^* change from -0.30 to $+0.30$, the calculated resluts are drawn on Figs. 4, 5, and 6. It is easily seen from the composite charts of equilibria from Figs. 4, 5, and 6 that the influences of the differences of the truncated spectral models on the solution of (1) is very important, and it warns us of carelessness utilizing a highly truncated low-spectral model to approximate a partial differential equation.

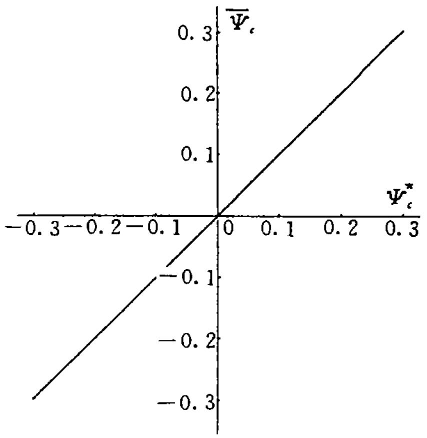

Fig. 5. The same as Fig. 4 except with 5 and 3 basic functions truncated.

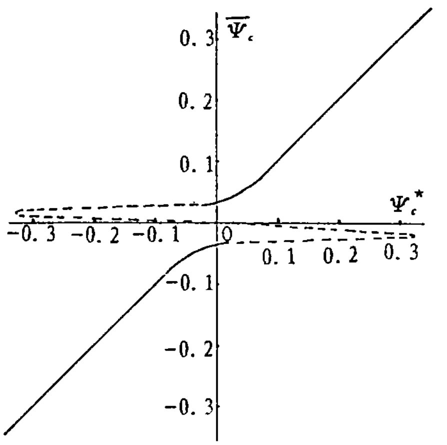

Fig. 6. The same as Fig. 4 except with 4 functions truncated.

V. CONCLUSIONS

It is shown from the calculated results above that by using a finite spectral model to approximate a nonlinear partial differential equation the solving process becomes simple, and the physical meanings of the solution are clear. These are its advantages. However, to obtain an appropriate solution one has to be very careful in choosing the approximate truncated spectral model, because the solutions of the chosen finite ordinary differential equations are not likely to converge to the true solutions of the partial differential equation. In addition, how many truncated spectra are appropriate to describe the physical phenomena which are contained in the partial differential equation, or can the chosen truncated spectra contain the primary parts of the physical phenomena? If the chosen truncated spectral model could not describe the primary features of the physical phenomena it is obvious that their true features which are desirable to be examined could not be obtained by using this spectral model.

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