INSTABILITY OF PERTURBATION IN TYPHOONS UNDER THE CONDITIONS OF APPROXIMATE ACTUAL WIND FIELD

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ABSTRACT

With conditions of approximately actual background wind field in typhoons and a barotropic nondivergent model in a cylindrical coordinate, the problem of instability of the perturbation in typhoons is discussed. The results show that the warm core structure of typhoons and the activity of fairly strong cold air in the periphery are advantageous to the perturbation development in typhoons, and the stronger the typhoons, i.e. the lower the pressure, the easier the perturbation in typhoons would develop.

Key words: typhoon, warm core structure, development of perturbation

I. INTRODUCTION

Theoretical research on the genesis and development of typhoon has been one of the important subjects of the general community of meteorologists. A theory of "thermal engine" was put forward by Riehl (1954), followed by a "prospective theory" (Li, 1956) that put stress on the role of cold air current making hemispheric cross at the equator to contribute essentially to the activity introduced above. Charney (1964) formulated his theory of CISK to tackle the issue. Quite recently, Wei (1988) completed his "multi-scale composite theory" for the issue of typhoon genesis with results by analysis of observations and modeling experiments. Apart from this work, Li (1983) discussed the effects of environmental flow field on the genesis and development of typhoon, and Fei and Lu (1996) studied the problem of instability due to inertial internal gravity wave and asymmetry, the latter instability was revealed to be another mechanism by which the typhoon forms and grows.

In nature, the typhoon is a cyclonic vortex that evolves around a warm core due to the heating by both the release of latent heat and downward motion over the eye region. The establishment of the warm-core structure is a significant feature as well as a key in the genesis and development of the typhoon by reducing the surface pressure and intensifying the cyclonic rotation.

For the typhoon in the South China Sea in spring and autumn, the relationship between the genesis and development and activity of cold air has been a major concern for Chinese meteorologists (Wei, Wang and Guo et al., 1965; Chen and Ding, 1979), but there has not been any complete agreement in the view of whether the former is associated with the enchancement and weakening of the typhoon.

In the previous treatment of the dynamic aspect of this problem, a usual form of environmental flow field (e.g. $\overline{V}=\Omega \cdot r$) used to be taken for simplicity, though it much deviated from the actual distribution. Taking an environmental flow field that is approximately consistent with the reality, the barotropic non-divergent model is used in this paper to study the genesis and the stability of development and the effects of the warm core and cold air current on the typhoon.

Ⅱ. MATHEMATICAL MODEL

With linearization, the barotropic non-divergent perturbation equation set is expressed as

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\overline{V}}{r}\frac{\partial u}{\partial \theta} - \tilde{f} v = -\frac{1}{\rho}\frac{\partial p}{\partial r} & \text{(1a)} \\ \frac{\partial v}{\partial t} + \frac{\overline{V}}{r}\frac{\partial v}{\partial \theta} + \hat{f} u = -\frac{1}{\rho}\frac{\partial p}{r\partial \theta} & \text{(1b)} \\ \frac{\partial ru}{\partial r} + \frac{\partial v}{\partial \theta} = 0, & \text{(1c)} \end{cases}$$

where the " ' " is omitted. Specifically,

$$\tilde{f} = f + \frac{2\overline{V}}{r}, \hat{f} = \frac{\overline{V}}{r} + \frac{\partial \overline{V}}{\partial r} + f.$$

In the eye region (called Region 1), the environmental wind field is assumed to be positively proportional with r, or,

$$\overline{V}_1 = \Omega r, \qquad (0 \leqslant r \leqslant r_0)$$

where Ω is the angular velocity of rotation in the environmental field and r_0 the radius of the eye.

In the region outside the eye (called Region 2), the environmental field is assumed to be negatively proportional with r, or,

$$\overline{V}_2 = \frac{M}{r}. \qquad (r_0 \leqslant r \leqslant \infty)$$

Following that $\overline{V}$ is continuous at r_0, it is known that $M = \Omega r_0^2$. The distribution of the environmental wind field $\overline{V}$ in the typhoon is seen in Fig. 1. The radial profile section of observed typhoon wind velocity is seen in Fig. 2. Obviously, the environmental wind field of the typhoon assumed here is very close to that in reality.

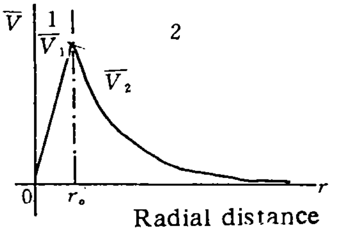

Fig. 1. Schematic of radial distribution of the assumed environmental wind field $\overline{V}$ of a typhoon.

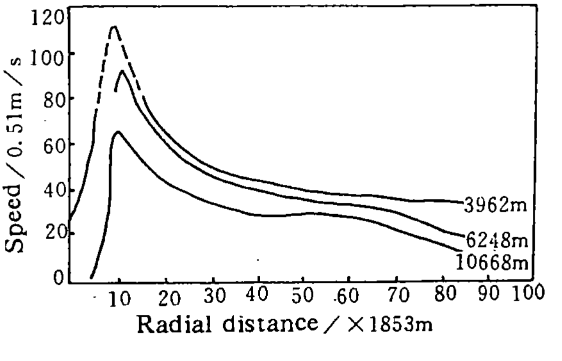

Fig. 2. Cross-section diagram of wind speed at three-height layer in hurricane Daisy in 1958.

In the discussion that follows, Ω and f are both taken as constant.

Ⅲ. ANALYTIC SOLUTION OF WIND AND PRESSURE FIELDS IN EYE REGION

Based on the assumption of $\overline{V}$ for Region 1, we have

$$\frac{\overline{V}_1}{r} = \Omega = \text{constant},$$
$$\widetilde{f}_1 = f + 2\Omega = \text{constant},$$
$$\hat{f}_1 = f + 2\Omega = \text{constant}.$$

Then, Eq. (1) for Region 1 becomes

$$\begin{cases} \dfrac{\partial u_1}{\partial t} + \Omega \dfrac{\partial u_1}{\partial \theta} - \widetilde{f}_1 v_1 = -\dfrac{1}{\rho_1}\dfrac{\partial p_1}{\partial r} & \text{(2a)} \\ \dfrac{\partial v_1}{\partial t} + \Omega \dfrac{\partial v_1}{\partial \theta} + \widetilde{f}_1 u_1 = -\dfrac{1}{\rho_1}\dfrac{\partial p_1}{r\partial \theta} & \text{(2b)} \\ \dfrac{\partial r u_1}{\partial r} + \dfrac{\partial v_1}{\partial \theta} = 0, & \text{(2c)} \end{cases}$$

where the subscript "1" denotes the eye region.

Let $\zeta = \ln \frac{r}{r_0}$, then $\zeta \rightarrow -\infty$ as $r \rightarrow 0$; $\zeta \rightarrow r_0$ as $r \rightarrow r_0$. And

$$d\zeta = \frac{1}{r} dr.$$

Consequently, Eq. (2) is reduced to

$$\begin{cases} \dfrac{\partial u_1}{\partial t} + \Omega \dfrac{\partial u_1}{\partial \theta} - \widetilde{f}_1 v_1 = -\dfrac{1}{\rho_1}\dfrac{1}{r}\dfrac{\partial p_1}{\partial \zeta} & \text{(3a)} \\ \dfrac{\partial v_1}{\partial t} + \Omega \dfrac{\partial v_1}{\partial \theta} + \widetilde{f}_1 u_1 = -\dfrac{1}{\rho_1}\dfrac{1}{r}\dfrac{\partial p_1}{\partial \theta} & \text{(3b)} \\ \dfrac{1}{r}\dfrac{\partial r u_1}{\partial \zeta} + \dfrac{\partial v_1}{\partial \theta} = 0. & \text{(3c)} \end{cases}$$

Let $U_1 = r u_1, V_1 = r v_1, P_1 = p_1$. Therefore, Eq. (3) can be written as

$$\begin{cases} \dfrac{\partial U_1}{\partial t} + \Omega \dfrac{\partial U_1}{\partial \theta} - \widetilde{f}_1 V_1 = -\dfrac{1}{\rho_1}\dfrac{\partial P_1}{\partial \zeta} & \text{(4a)} \\ \dfrac{\partial V_1}{\partial t} + \Omega \dfrac{\partial V_1}{\partial \theta} + \widetilde{f}_1 U_1 = -\dfrac{1}{\rho_1}\dfrac{\partial P_1}{\partial \theta} & \text{(4b)} \\ \dfrac{\partial U_1}{\partial \zeta} + \dfrac{\partial V_1}{\partial \theta} = 0. & \text{(4c)} \end{cases}$$

Let

$$U_1 = i\hat{U}_1(\zeta)e^{i(\omega t - m\theta)},$$
$$V_1 = \hat{V}_1(\zeta)e^{i(\omega t - m\theta)},$$
$$P_1 = \hat{P}_1(\zeta)e^{i(\omega t - m\theta)},$$

where m is the angular wavenumber in the direction of θ and by taking $m \geqslant 0$ and substituting into Eq. (4), we have

$$\begin{cases} -(\omega - m\Omega)\hat{U}_1 - \tilde{f}_1\hat{V}_1 = -\dfrac{1}{\rho_1}\dfrac{d\hat{P}_1}{d\zeta} & \text{(5a)} \\ (\omega - m\Omega)\hat{V}_1 + \tilde{f}_1\hat{U}_1 = -\dfrac{1}{\rho_1}m\hat{P}_1 & \text{(5b)} \\ \dfrac{d\hat{U}_1}{d\zeta} - m\hat{V}_1 = 0, & \text{(5c)} \end{cases}$$

which is reduced to a univariable equation of

$$(\omega - m\Omega)\frac{d^2\hat{U}}{d\zeta^2} - m^2(\omega - m\Omega)\hat{U}_1 = 0 \tag{6}$$

and as $\omega - m\Omega \neq 0$, we have

$$\frac{d^2\hat{U}_1}{d\zeta^2} - m^2\hat{U}_1 = 0. \tag{7}$$

As a result, solutions can be derived as

$$\hat{U}_1 = Ae^{m\zeta} + Be^{-m\zeta}.$$

Since $\hat{U}_1$ shows boundedness as the boundary condition $\zeta \to -\infty (r \to 0)$, only when $B = 0$ can we have

$$\hat{U}_1 = Ae^{m\zeta}. \tag{8}$$

It is obtained from (5c) and (5b) that

$$\hat{V}_1 = Ae^{m\zeta}, \tag{9}$$

$$\hat{P}_1 = \frac{\rho_1}{m}(\omega - m\Omega + 2\Omega + f)Ae^{m\zeta}, \tag{10}$$

and further on the analytic solution to the wind and pressure fields in Region 1 can be obtained as

$$u_1 = \frac{1}{r}U_1 = -A\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{m-1}\sin(\omega t - m\theta), \tag{11}$$

$$v_1 = \frac{1}{r}V_1 = A\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{m-1}\cos(\omega t - m\theta), \tag{12}$$

$$p_1 = P_1 = \frac{\rho_1 A}{m}(\omega - m\Omega + 2\Omega + f)\left(\frac{r}{r_0}\right)^{m}\cos(\omega t - m\theta). \tag{13}$$

Ⅳ. ANALYTIC SOLUTION OF WIND AND PRESSURE FIELDS OUTSIDE THE EYE REGION

For Region 2,

$$\frac{\overline{V}_2}{r} = \frac{\Omega r_0^2}{r^2}$$

$$\tilde{f}_2 = f + \frac{2\Omega r_0^2}{r^2}$$

$$\hat{f}_2 = f$$

are substituted into Eq. (1) to obtain

$$\begin{cases} \frac{\partial u_2}{\partial t} + \frac{\Omega r_0^2}{r^2}\frac{\partial u_2}{\partial \theta} - (f + \frac{2\Omega r_0^2}{r^2})v_2 = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial r} & (14a) \\ \frac{\partial v_2}{\partial t} + \frac{\Omega r_0^2}{r^2}\frac{\partial v_2}{\partial \theta} + f u_2 = -\frac{1}{\rho_2}\frac{\partial p_2}{r\partial \theta} & (14b) \\ \frac{\partial r u_2}{\partial r} + \frac{\partial v_2}{\partial \theta} = 0. & (14c) \end{cases}$$

The subscript "2" stands for the outer part of the eye region. Let $U_2 = ru_2, V_2 = rv_2$, and $P_2 = p_2$, and substitute them into Eq. (14), then we have

$$\begin{cases} \frac{\partial U_2}{\partial t} + \frac{\Omega r_0^2}{r^2}\frac{\partial U_2}{\partial \theta} - (f + \frac{2\Omega r_0^2}{r^2})V_2 = -\frac{r}{\rho_2}\frac{\partial P_2}{\partial r} & (15a) \\ \frac{\partial V_2}{\partial t} + \frac{\Omega r_0^2}{r^2}\frac{\partial V_2}{\partial \theta} + f U_2 = -\frac{1}{\rho_2}\frac{\partial P_2}{\partial \theta} & (15b) \\ r\frac{\partial U_2}{\partial r} + \frac{\partial V_2}{\partial \theta} = 0. & (15c) \end{cases}$$

Let

$$U_2 = i\hat{U}_2(r)e^{i(\omega t - m\theta)}$$

$$V_2 = \hat{V}_2(r)e^{i(\omega t - m\theta)}$$

$$P_2 = \hat{P}_2(r)e^{i(\omega t - m\theta)}.$$

and substitute them into (15) to derive

$$\begin{cases} -(\omega - m\Omega\frac{r_0^2}{r^2})\hat{U}_2 - (f + \frac{2\Omega r_0^2}{r^2})\hat{V}_2 = -\frac{r}{\rho_2}\frac{d\hat{P}_2}{dr} & (16a) \\ (\omega - m\Omega\frac{r_0^2}{r^2})\hat{V}_2 + f\hat{U}_2 = \frac{m}{\rho_2}\hat{P}_2 & (16b) \\ \hat{V}_2 - \frac{r}{m}\frac{d\hat{U}_2}{dr} = 0, & (16c) \end{cases}$$

which is then reduced to a univariable equation of

$$\frac{r^2}{m}(\omega - m\Omega\frac{r_0^2}{r^2})\frac{d^2\hat{U}}{dr^2} + \frac{r}{m}(\omega - m\Omega\frac{r_0^2}{r^2})\frac{d\hat{U}_2}{dr} - m(\omega - m\Omega\frac{r_0^2}{r^2})\hat{U}_2 = 0. \quad (17)$$

When $\omega - m\Omega\frac{r_0^2}{r^2} \neq 0$,

$$r^2\frac{d^2\hat{U}_2}{dr^2} + r\frac{d\hat{U}_2}{dr} - m^2\hat{U}_2 = 0 \quad (18)$$

with its solution being

$$\hat{U}_2 = Ar^m + Br^{-m}. \quad (19)$$

In view of the fact that $\hat{U}_2$ is bounded when $r = r_0$ and $r \to \infty$, then $A = 0$. As a result,

$$\hat{U}_2 = Br^{-m} \quad (20)$$

It is obtained from Eqs. (16c) and (16b) that

$$\hat{V}_2 = -Br^{-m}, \quad (21)$$

$$\hat{P}_2 = \frac{\rho_2}{m}Br^{-m}(-\omega + m\Omega\frac{r_0^2}{r^2} + f). \quad (22)$$

Then, the analytic solutions of wind and pressure fields in Region 2 becomes

$$u_2 = \frac{1}{r}U_2 = -Br^{-(m+1)}\sin(\omega t - m\theta), \quad (23)$$

$$v_2 = \frac{1}{r}V_2 = -Br^{-(m+1)}\cos(\omega t - m\theta), \quad (24)$$

$$p_2 = P_2 = \frac{\rho_2}{m}B(f - \omega + m\Omega\frac{r_0^2}{r})r^{-m}\cos(\omega t - m\theta). \quad (25)$$

V. FREQUENCY EQUATIONS

Following the dynamic condition that the pressure is continuous at r_0, we have

$$\begin{cases} \frac{\partial}{\partial t}(p_1 - p_2) + u_1\frac{\partial}{\partial r}(\bar{p}_1 - \bar{p}_2) + \frac{\bar{V}_1}{r}\frac{\partial}{\partial\theta}(p_1 - p_2)]_{r=r_0} = 0 & (26a) \\ \frac{\partial}{\partial t}(p_1 - p_2) + u_2\frac{\partial}{\partial r}(\bar{p}_1 - \bar{p}_2) + \frac{\bar{V}_2}{r}\frac{\partial}{\partial\theta}(p_1 - p_2)]_{r=r_0} = 0. & (26b) \end{cases}$$

As the environmental field satisfies the relation of gradient winds as in

$$f\bar{V} + \frac{\bar{V}^2}{r} = \frac{1}{\rho}\frac{d\bar{p}}{dr},$$

then,

$$\frac{\mathrm{d}\overline{p}_1}{\mathrm{d}r}\Big|_{r=r_0} = \rho_1 R_0,$$
$$\frac{\mathrm{d}\overline{p}_2}{\mathrm{d}r}\Big|_{r=r_0} = \rho_2 R_0,$$

where $R_0 = (f + \Omega)\Omega r_0$, therefore

$$\frac{\partial}{\partial r}(\overline{p}_1 - \overline{p}_2)\Big|_{r=r_0} = (\rho_1 - \rho_2)R_0. \tag{27}$$

From Eqs. (13) and (25), we have

$$\frac{\partial}{\partial t}(p_1 - p_2)\Big|_{r=r_0} = -\frac{\omega}{m}(\rho_1 A\varepsilon_1 - \rho_2 B\varepsilon_2)\sin(\omega t - m\theta), \tag{28}$$
$$\frac{\partial}{\partial \theta}(p_1 - p_2)\Big|_{r=r_0} = (\rho_1 A\varepsilon_1 - \rho_2 B\varepsilon_2)\sin(\omega t - m\theta), \tag{29}$$

where $\varepsilon_1 = \omega + 2\Omega - m\Omega + f$, $\varepsilon_2 = (f - \omega + m\Omega)r_0^{-m}$.

Substituting Eqs. (27) ~ (29) into Eq. (26) and taking into account that $\overline{V}_1 = \overline{V}_2 = \overline{V}_0$ at $r = r_0$, we have

$$\begin{cases} [-\rho_1\varepsilon_1(\frac{\omega}{m} - \frac{\overline{V}_0}{r}) - \frac{1}{r_0}(\rho_1 - \rho_2)R_0]A + (\frac{\omega}{m} - \frac{\overline{V}_0}{r_0})\rho_2\varepsilon_2 B = 0, & (30a) \\ -(\frac{\omega}{m} - \frac{\overline{V}_0}{r_0})\rho_1\varepsilon_1 A + [\rho_2\varepsilon_2(\frac{\omega}{m} - \frac{\overline{V}_0}{r_0}) - (\rho_1 - \rho_2)R_0 r^{-(m+1)}]B = 0. & (30b) \end{cases}$$

To make the above set of equations solvable, its system determinant should be set at zero. If $\frac{\omega}{m} - \frac{\overline{V}_0}{r_0} \neq 0$ and $\psi = \frac{\omega}{m} - \frac{\overline{V}_0}{r_0}$ is assumed, then

$$\begin{vmatrix} -\rho_1\varepsilon_1 - \dfrac{R_0(\rho_1 - \rho_2)}{r_0\psi} & \rho_2\varepsilon_2 \\ -\rho_1\varepsilon_1 & \rho_2\varepsilon_2 - \dfrac{R_0(\rho_1 - \rho_2)}{\psi}r_0^{-(m+1)} \end{vmatrix} = 0.$$

If $\rho_1 - \rho_2 \neq 0$, the determinant above can be written as

$$\rho_1\varepsilon_1 r_0\psi - \rho_2\varepsilon_2 r_0^{(m+1)}\psi - R_0(\rho_1 - \rho_2) = 0 \tag{31}$$

and substitute it with expressions for ε_1、ε_2 and ψ and we have

$$\frac{\rho_1 + \rho_2}{m}\omega^2 + \{\frac{1}{m}[\rho_1(2\Omega - m\Omega + f) - \rho_2(f + m\Omega)]$$
$$- (\rho_1 + \rho_2)\frac{\overline{V}_0}{r_0}\}\omega - \frac{\overline{V}_0}{r_0}[\rho_1(2\Omega - m\Omega + f) - \rho_2(f + m\Omega)]$$
$$- (f + \Omega)\Omega(\rho_1 - \rho_2) = 0 \tag{32}$$

so that

$$\omega_{1,2}=-\frac{1}{2}(C_0-\frac{\overline{V}_0}{r_0}m)\pm\frac{1}{2}\sqrt{(C_0+\frac{\overline{V}_0}{r_0}m)^2+\frac{\rho_1-\rho_2}{\rho_1+\rho_2}m\Omega(f+\Omega)}, \quad (33)$$

where $C_0=\frac{\rho_1}{\rho_1+\rho_2}(2\Omega-m\Omega+f)-\frac{\rho_2}{\rho_1+\rho_2}(m\Omega+f)$. It is obvious that the frequency ω is related to Ω, m, f, ρ_1, ρ_2, $\overline{V}_0$ and r_0. Only when $\rho_1<\rho_2$, i.e. $\rho_2-\rho_1>\frac{(\rho_1+\rho_2)(C_0+\frac{\overline{V}_0}{r_0}m)^2}{m\Omega(f+\Omega)}$, can the sum of all terms within the radical sign be less than zero, or, the disturbance in the typhoon becomes instable. If $\rho_1\approx\rho_2$, it is unlikely that the disturbance becomes instable.

VI. CONCLUDING REMARKS

a. One of the important prerequesites for the disturbance to grow in the typhoon is the warm-core structure.

The principal feature is the warm core concerning the thermal property of typhoon. The structure appears in the stage of development and grows to the maximum in the mature stage. Once it weakens and disappears, the typhoon will transform in nature and weaken or fill up. The stronger the structure, the greater the difference in density between the eye region and the outward region will be. The intensity of the warm core in typhoon is a measure for determining whether the perturbation inside grows or not. The stronger the structure, the more easily the perturbation will develop.

b. The activity of environmental cold air with adequate degree of intensity is also favourable for the growth of the in-typhoon perturbation.

Difference in the view of the relationship between the cold air activity and the change of typhoon intensity has long existed. Some argue that the cold current enchances the typhoon while others try to justify that it weakens the storm (Chen et al., 1979). It is shown in this study that the cold air outside the typhoon increases the difference in density between the eye region and the environment and thus enables it to develop, which agrees with the findings by the observations for the typhoon in the South China Sea in spring and autumn (Wei et al., 1965). When the cold air is strong enough to destroy the warm core, the difference will be less obvious so that the typhoon will decay or dissipate completely. If, on the other hand, the cold air is so weak that it does not differ much from that previously existent in the low latitudes, it will have limited effects on the typhoon.

c. The greater the pressure difference between the eye region and the environment, the more likely the perturbation in typhoon will be to grow.

The difference in pressure between the eye region and the environment is another measure reflecting that in density. Therefore, a stronger typhoon (a lower central pressure) means greater possibility for the perturbation to grow. It is more unlikely that it will do so if the typhoon weakens or fills up, a phenomenon that is consistent with observations.

The activity of cold air outside the typhoon also increases the difference in pressure between the eye region and the environment. It is from this point of view that the former is said to have positive effects for the development of perturbation in typhoon.

With an environmental field that is close to the reality, the results achieved here are definitely meaningful, though fall short of being thorough due to employment of a simple barotropic non-divergent model in the description of the life cycle of typhoon at sub-synoptic or even finer scales.

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