DEVELOPMENT OF A TROPICAL CYCLONE IN BAROTROPIC ENVIRONMENTAL FLOWS

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ABSTRACT

Development of a tropical cyclone in barotropic environmental flows is investigated with a shallow water model. It is found that the tropical cyclone develops only when it is embedded in an environmental flow with a southward relative vorticity gradient. The related physical mechanism is explained by analyzing the kinetic energy conversion between the environmental flow and tropical cyclone circulation.

Key words: barotropic environmental flows, tropical cyclone, kinetic energy conversion, relative vorticity gradient.

Ⅰ. INTRODUCTION

There have been a lot of research which contributed to the tropical cyclone propagation. In a resting environment, a tropical cyclone drifts due to the advection of axially symmetric vorticity by axially asymmetric circulation, which results from the advection of planetary vorticity by axially symmetric circulation of the tropical cyclone (β effect). The drift speed and direction depend on the horizontal and vertical structures of the tropical cyclones (e. g. , Holland 1983; Chan and Williams; Fiorino and Elsberry 1989; Wang and Li 1992; and Li and Wang 1994), and horizontal structures of environmental flows (e. g. , Wang and Li 1995; and Li and Wang 1996). In the present study, we will investigate development of a tropical cyclone embedded in barotropic environmental flows with a shallow water model. Section 2 describes environmental flows and the model. Section 3 examines which type of environmental flow has strong impact on the development of the tropical cyclone. Section 4 explains the related physical mechanism. We summarize the major findings in the last section.

Ⅱ. ENVIRONMENTAL FLOWS AND NUMERICAL MODEL

Li and Wang (1996) calculated the vertically averaged wind between 300 and 850 hPa derived from the European Center for Medium-range Weather Forecast analysis for the period of 10～17 July 1987, which is shown in Fig. 1. Fig. 1 shows that the mean flow consists of a subtropical ridge around 28°N, a monsoon trough to its south and a westerly trough to its north. We derived meridional variation of the zonal wind along line AB and zonal variation of the meridional wind along line CD in Fig. 1 (Fig. 2). The zonal wind has strong easterly whereas the meridional wind has strong northerly. Since divergent wind is one order of magnitude smaller than rotational wind, divergent wind is negligible. Thus the environmental flow ($\vec{U}_e$) can be expressed as, in the Cartesian coordinates,

$$\vec{U}_e = \vec{i}U(y) + \vec{j}V(x) \tag{1}$$

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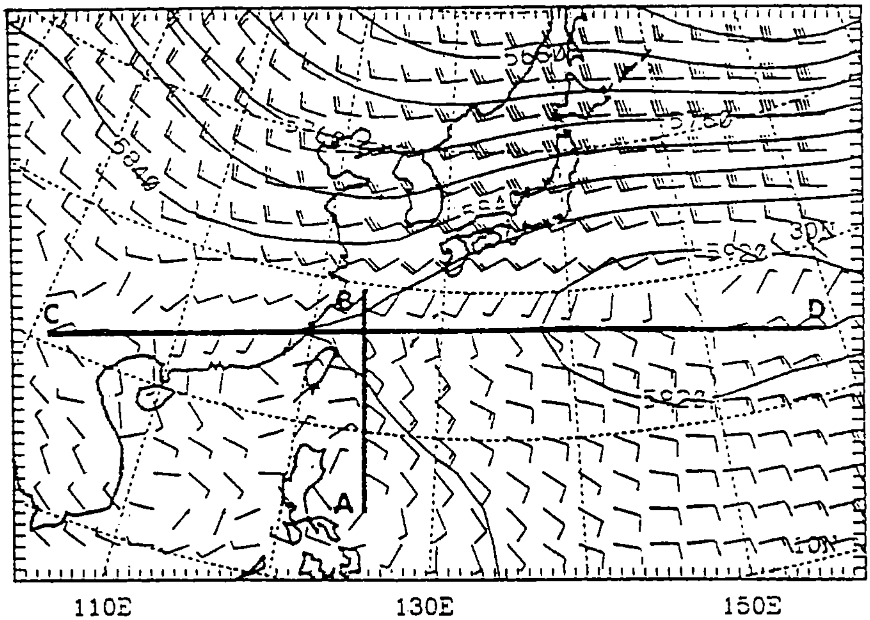

Fig. 1. Vertical mean wind geopotential height (contour) calculated by averaging for 300, 400, 500, 700 and 850 hPa data for the period of 10-27 July 1987. The data used is from the analysis made by the European Centre for Medium-range Weather Forecast. Wind with full barb represents 5 ms^{-1}. The interval of geopotential height is 40 m.

Where U and V are the zonal and meridional components of the environmental flow, respectively; $\vec{i}$ and $\vec{j}$ are the unit vectors, respectively, in the zonal and meridional directions. Taking a Taylor series expansion for the zonal and meridional winds around an arbitrary point (x_0, y_0), we obtain

$$U(y) = U_0 + U_y(y - y_0) + 0.5U_{yy}(y - y_0)^2 + \cdots, \tag{2}$$
$$V(x) = V_0 + V_x(x - x_0) + 0.5V_{xx}(x - x_0)^2 + \cdots, \tag{3}$$

where the first, second and third term in the right-hand side are uniform, linear and quadratic component, respectively; U_0, V_0, U_y, V_x, U_{yy}, V_{xx} are constants. The uniform wind does not interact with cyclone and linear wind has no significant effect on strength of tropical cyclone (Wang and Li 1995), whereas quadratic wind affects the strength of cyclone significantly. Only quadratic wind is thus considered in the study.

A shallow water model is used in the study. The detailed model description can be found in Wang and Li (1995). Wang and Li (1995) demonstrated that the variations of the environmental flow have no significant impact on tropical cyclone circulation when environmental flow is the steady solution of the model. We fixed the environmental flow during the integration. The initial tropical cyclone circulation is axially symmetric. Its maximum wind is 30 ms^{-1} at radius of 100 km, and its cyclonic circulation extends to 750 km (Fig. 3).

An energetics analysis is an effective way to study how the environmental flow affects the development of tropical cyclone. Wang and Li (1995) derived a set of kinetic energy equations by decomposing total wind into environmental flow, axially symmetric

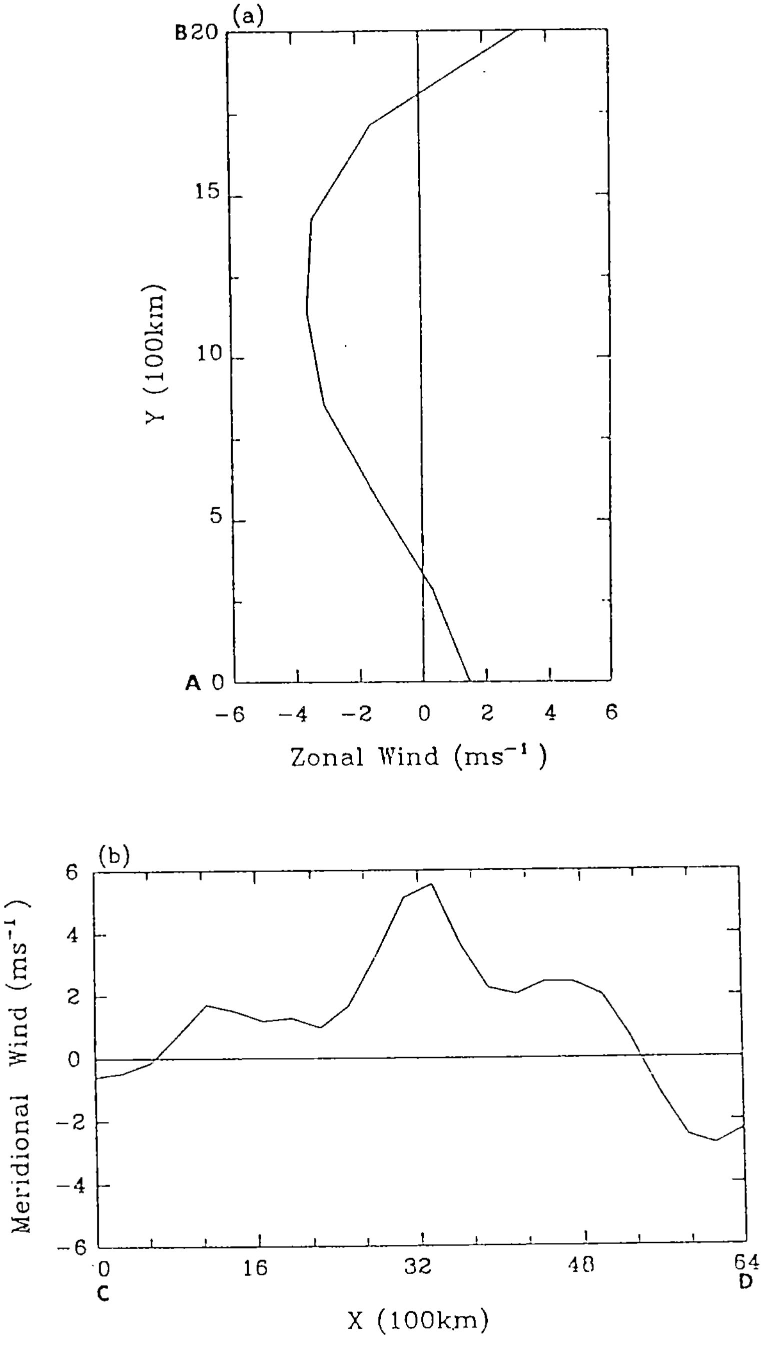

Fig. 2. (a) Meridional variation of zonal wind and (b) zonal variation of meridional wind along lines AB and CD, respectively, in Fig. 1.

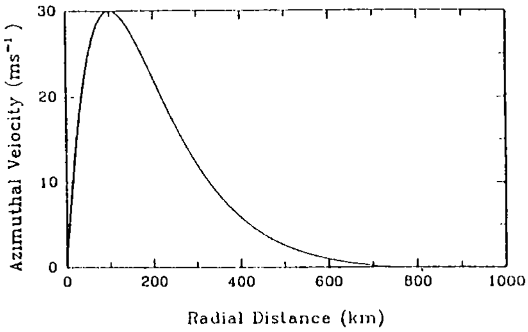

Fig. 3. Azimuthal wind profile of the initial symmetric circulation of tropical cyclone.

and axially asymmetric azimuthal wavenumber-one (beta-gyre) and residual components of tropical cyclone circulation:

$$\frac{\partial K_s}{\partial t} = F_s + (K_g, K_s) + (K_{res}, K_s) + (K_e, K_s), \tag{4}$$

$$\frac{\partial K_g}{\partial t} = F_g + (K_s, K_g) + (K_{res}, K_g) + (K_e, K_g), \tag{5}$$

$$\frac{\partial K_{res}}{\partial t} = F_{res} + (K_s, K_{res}) + (K_g, K_{res}) + (K_e, K_{res}), \tag{6}$$

where K and F are kinetic energy and energy flux along lateral boundary, respectively; subscripts "s", "g", "res" represent, respectively, axially symmetric and axially asymmetric azimuthal wavenumber-one and residual components of tropical cyclone circulation. The detailed derivation of equations and energy conversion terms can be found in Wang and Li (1995). Since the effect of interaction between the environmental flow and tropical cyclone on development of tropical cyclone is examined in this study, and most of kinetic energy of tropical cyclone circulation is contributed by axially symmetric component (also see Table 1 and discussions in the next section), the energy conversion

Table 1. Total kinetic energy of tropical cyclone K, K_s and kinetic energy conversion terms (K_g, K_s), (K_{res}, K_s), (K_e, K_s) and $\frac{\partial K_s}{\partial t}$ at hour 96 for five cases. The units of kinetic energy and its conversion are $10^{12}\mathrm{m^4s^{-2}}$ and $10^7\mathrm{m^4s^{-3}}$, respectively.

Case	K	K_s	(K_g, K_s)	(K_{res}, K_s)	(K_e, K_s)	$\frac{\partial K_s}{\partial t}$
Q_0	73.3	65.0	−2.7	0.2	0.0	−2.5
Q_1	92.0	83.0	−5.0	−0.7	11.4	5.7
Q_2	65.7	57.2	−2.8	−1.7	−0.1	−4.5
Q_3	69.6	61.1	−1.5	0.7	−1.7	−2.5
Q_4	74.0	66.3	−2.9	0.4	0.7	−1.8

term (K_e, K_s) will be analyzed.

$$(K_e, K_s) = -\langle [(\vec{U}_s + \vec{U}_g + \vec{U}_{res}) \cdot \nabla \vec{U}_e] \cdot \vec{U}_s \rangle \tag{7}$$

where $\vec{U}$ is a total wind vector in Cartesian coordinates, angular bracket

$$\langle(\ )\rangle = \iint_S (\ )\mathrm{d}S = \iint_{A(t)} (\ )\mathrm{d}A = \int_0^R r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\lambda \tag{8}$$

represents an integration over the model domain S. Because of the circular nature of the cyclone circulation, the integration of kinetic energy and related conversion terms over the model domain (S) can be approximately obtained by integrations over a circular domain $A(t)$ whose radius is R and whose center is collocated with the center of moving cyclone. For the convenience of analysis, the integration can be carried out in local cylindrical coordinates whose origin is collocated with the cyclone center. In local cylindrical coordinates. Eq. (8) becomes

$$\begin{aligned}(K_e,K_s) = &- \langle[(\vec{V}_s + \vec{V}_g + \vec{V}_{res})\cdot\nabla\vec{V}_e]\cdot\vec{V}_s\rangle - \\ &\langle(\Omega_s + \Omega_g + \Omega_{res})\vec{k}\cdot(\vec{V}_e\times\vec{V}_s)\rangle,\end{aligned} \tag{9}$$

where $\vec{V}$ is a total wind vector in local cylindrical coordinates; $\Omega = v_\lambda r^{-1}$, v_λ is a azimuthal wind component.

Ⅲ. DEVELOPMENT OF A TROPICAL CYCLONE IN BAROTROPIC ENVIRONMENTAL FLOWS

Five experiments with different quadratic environmental flows were carried out. Case Q_0 has a resting environment. Cases Q_1 and Q_2 have β and $-\beta$ of U_{yy}, respectively, whereas Cases Q_3 and Q_4 have $-\beta$ and β of V_{xx}, respectively. β is meridional gradient of Earth vorticity at 20°N.

The energy budget calculations at hour 96 show the kinetic energy of cyclone circulation K is most contributed by symmetric cyclone circulation K_s for all five cases because the ratio of K_s to K is about 0. 87～0. 9. K_s is the largest in case Q_1 and it is smallest in case Q_2 among five cases. The energy conversion terms (K_g, K_s) are negative for all five cases, indicating that kinetic energy is converted from symmetric calculation to beta gyres. The conversion term (K_{res}, K_s) are also negative in cases Q_1 and Q_2, whereas they are positive with magnitudes less than those of (K_g, K_s) in other three cases. Thus the development of tropical cyclone depends on the energy conversion term (K_e, K_s). In case Q_1, (K_e, K_s) is positive with a large magnitude. It is large positive (K_e, K_s) that causes positive $\frac{\partial K_s}{\partial t}$ and maximum K_s. Therefore, it is necessary to examine the energy conversion term (K_e, K_s).

Ⅳ. EXPLANATION

When a cyclone is centered at $x = x_c, y = y_c$ at time $t = t_c$, the quadratic environmental flow $\vec{U}_e$ could be

$$\begin{aligned}\vec{U}_e = &\vec{i}U_{yy}(y_c - y_0)^2 + \vec{j}V_{xx}(x_c - x_0)^2 + \\ &\vec{i}U_{yy}(y_c - y_0)(y - y_0) + \vec{j}V_{xx}(x_c - x_0)(x - x_0) + \\ &\vec{i}U_{yy}(y - y_c)^2 + \vec{j}V_{xx}(x - x_c)^2,\end{aligned} \tag{10}$$

where the first and second term of the right-hand side of (10) represent a uniform component; the third and fourth term of the the right-hand side of (10) denote a linear component; the fifth and sixth term of the the right-hand side of (10) represent a quadratic component, which has the same form as the third terms of the right-hand side of (2) except (x_0, y_0) is replaced by (x_c, y_c). The results indicate that only the quadratic component has significant impact on conversion term (K_e, K_s). Thus,

$$U_e = U_{yy}(y - y_c)^2, \tag{11}$$
$$V_e = V_{xx}(x - x_c)^2. \tag{12}$$

In local cylindrical coordinates, the environmental flow in cases Q_1 and Q_2 is

$$V_{re} = - U_e \sin\lambda = - 0.125 U_{yy} r^2 (\sin\lambda + \sin 3\lambda), \tag{13}$$
$$V_{\lambda e} = - U_e \cos\lambda = - 0.125 U_{yy} r^2 (3\cos\lambda + \cos 3\lambda), \tag{14}$$

and the environmental flow in cases Q_3 and Q_4 is

$$V_{re} = - V_e \cos\lambda = 0.125 V_{xx} r^2 (\cos\lambda - \cos 3\lambda), \tag{15}$$
$$V_{\lambda e} = - V_e \sin\lambda = 0.125 V_{xx} r^2 (-3\sin\lambda + \sin 3\lambda). \tag{16}$$

where V_{re} and $V_{\lambda e}$ are azimuthal and radial wind components, respectively, in local cylindrical coordinates. Using (13), (14), (15) and (16) respectively, in (7) and neglecting the effect of residual asymmetric flow yields

$$(K_e, K_s) = U_{yy} \langle r v_{\lambda s} \cos^2\lambda (v_{\lambda g}\cos\lambda - v_{\lambda g}\sin\lambda) \rangle \tag{17}$$

for cases Q_1 and Q_2, and

$$(K_e, K_s) = V_{xx} \langle r v_{\lambda s} \sin^2\lambda (v_{\lambda g}\sin\lambda + v_{\lambda g}\cos\lambda) \rangle \tag{18}$$

for cases Q_3 and Q_4.

As in Wang and Li (1995), ws assume that the streamfunction of the beta gyres can be approximately expressed by

$$\psi_g = R_g(r)\cos(\alpha - \lambda), \tag{19}$$

where R_g is the beta-gyre amplitude, and α is the azimuthal angle of the anticyclonic-gyre center measured counterclockwise from due north.

Substituting (19) into (17) and (18) leads to

$$(K_e, K_s) = U_{yy} \sin\alpha \left[\frac{\pi}{4} \int_0^R R_g r^3 \frac{\partial}{\partial r}\left(\frac{v_{\lambda s}}{r}\right) \mathrm{d}r \right] \tag{20}$$

for cases Q_1 and Q_2, and

$$(K_e, K_s) = - V_{xx} \cos\alpha \left[\frac{\pi}{4} \int_0^R R_g r^3 \frac{\partial}{\partial r}\left(\frac{v_{\lambda s}}{r}\right) \mathrm{d}r \right] \tag{21}$$

for cases Q_3 and Q_4.

Since the azimuthal angular wind $\left(\frac{v_{\lambda s}}{r}\right)$ decreases with increasing radial distance for the chosen cyclone profile and R_g is always positive, the value of the square bracket in (20) and (21) is negative. In cases Q_1 and Q_2, $\sin\alpha$ is negative (Wang and Li 1995). In case Q_1, $U_{yy} > 0$ and thus (K_e, K_s) is positive. In case Q_2, $U_{yy} < 0$ and thus (K_e, K_s) is negative. In cases Q_3 and Q_4, $\cos\alpha$ is positive. In case Q_3, $V_{xx} < 0$ and thus (K_e, K_s) is negative. In case Q_4, $V_{xx} > 0$ and thus (K_e, K_s) is positive. We compared case Q_1 with case Q_4 and found that the conversion rate of (K_e, K_s) in case Q_1 is much larger than that in case Q_4. From (20) and (21), it is found that (K_e, K_s) is proportional to $\sin\alpha$ in case Q_1, whereas it is proportional to $\cos\alpha$ in case Q_4. Since α in cases Q_1 and Q_4 is about 275~280°, the magnitude of $\sin\alpha$ is much larger than that of $\cos\alpha$. Therefore, the tropical cyclone in case Q_1 receives more kinetic energy than that in case Q_4.

V. SUMMARY

In this study, we investigate effect of interaction of barotropic environmental flows with a tropical cyclone on development of the tropical cyclone with a shallow water model. We found that only when it is embedded in an environmental flow with a southward relative vorticity gradient, the tropical cyclone develops. The kinetic energy budget analysis indicates that development of the tropical cyclone depends on kinetic energy exchange between its circulation and the environmental flow. It is the southward relative vorticity gradient of the environmental flow that converts large amount of kinetic energy into tropical cyclone circulation which in turn strengthens.

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