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# Comparison and Analysis of Two Different Calculation Schemes of SLEVE-Hybrid Coordinate in the GRAPES_MESO Model

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**Abstract:** The calculation scheme of the smoothed-level and hybrid (SLEVE-hybrid for short) coordinates in numerical forecasting model is not limited in number. It is divided into the semi-analytical scheme and the finite differential scheme in terms of the various differential methods of the coordinate deformation variables. Having compared the dynamic equation and the long-time batch simulation results of the two schemes, the present study draws the following conclusions. The first-order finite difference accuracy of the coordinate deformation variables in the finite differential scheme is theoretically lower than that in the semi-analytical scheme. The larger the vertical gradient of the layer thickness is, the larger the relative errors of the finite differential scheme are. The long-time batch simulation test in the GRAPES model dynamic core demonstrates that the bias of the temperature and the geopotential height in the semi-analytical scheme is smaller under the default layering, while the simulation difference of the two schemes is greatly reduced when the layering is more uniform.

**Key words:** GRAPES model; SLEVE-hybrid coordinate; dynamic core; numerical modelling
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## 1 INTRODUCTION

Terrain plays an important dynamical and thermal role in atmospheric movement, and the dynamical feedback of the terrain is crucial to the accuracy of numerical prediction. The terrain-following coordinate contains a lot of terrain information and has many advantages in numerical prediction models. A proper vertical coordinate can transform the complex control equations into simpler ones. Thereby, the lower boundary conditions are simple and easy to give. Meanwhile, it can effectively reduce various calculation errors and correctly describe the effect of terrain on the atmosphere.

To improve the simulation capabilities of numerical forecasting models, it is necessary to increase the horizontal resolution of the models. However, with the continuous improvement of the horizontal resolution, the terrain that a model can distinguish becomes steeper, and the slope of the vertical coordinate surfaces becomes larger and larger.

The GRAPES model, independently developed by the China Meteorological Administration, currently applies the traditional terrain_following coordinate (Gal-Chen[1]; Gal. C. S coordinate for short): $\hat{z} = Z_T \dfrac{z - Z_s}{Z_T - Z_s}$, where $\hat{z}$ represents the relative height of coordinate surfaces, $z$ represents the absolute height of coordinate surfaces, $Z_T$ represents mode top height and $Z_s$ represents terrain height. This form of coordinate allows model calculations to be performed in a "rectangular" bounded domain, which is convenient for computer programming and automatic calculation. It makes the irregular lower boundary in the actual atmosphere a smooth and undulating boundary surface in the model, which is easy to give the lower boundary conditions and describe the movement of the airflow along the terrain. The coordinate turns the smooth lower boundary surface to the upper boundary surface of the surface (land surface, and sea surface) boundary layer, which is convenient for the coupling of model dynamics, thermal processes and surface boundary layer parameterization (Phillps et al. [9]). However, the problem of the Gal. C. S coordinate is also very prominent. Terrain-following coordinate is orthogonal coordinate in a non-strict sense (Zdunkowski [14]). When coordinates transform from non-terrain-following ones to terrain-following ones, the pressure gradient calculation changes from one item to two items of comparable magnitude, bringing inevitable errors of pressure gradient force (PGF for short). The coordinate surfaces of the coordinate undulate with the terrain, and the uplifting coordinate surfaces in the lower layer will extend all the way to the top of the model layer. Although the slope of the coordinate surfaces decreases with the increase of altitude, the movement of the model atmosphere along the undulating model surfaces still has a large deviation

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from the quasi-horizontal movement of the real atmosphere. As the resolution of the GRAPES model improves, the terrain slope that can be recognized by the Gal. C. S coordinate increases, and the terrain gravity wave will be severely distorted.

The smoothed-level coordinate (SLEVE coordinate) proposed by *Schär* et al. [10] deformed the height terrain-following coordinate form and adjusted the terrain influences on the model surfaces by the terrain attenuation coefficient that changes with height. A designed experiment comparing the single-scale SLEVE1 coordinate and the dual-scale SLEVE2 coordinate was performed in the research. Li et al. [7] selected different terrain attenuation functions and conducted a series of theoretical analysis and ideal experiments. The experimental results show the advantages of SLEVE-hybrid coordinates in reducing PGF errors and advection dissipation is obvious. Based on the GRAPES model, Li et al. [4] selected the Gal. CS coordinate and the SLEVE1 coordinate to perform a simple case simulation. The SLEVE1 coordinate has smaller forecast noises. Li et al.[5] performed ideal tests and real batch simulations based on GRAPES model, selecting the Gal. C. S coordinate, SLEVE1 coordinate, SLEVE2 coordinate and a COS coordinate with cosine function as the attenuation basis function. Comparing various types of SLEVE-hybrid coordinates, it was found that SLEVE2 coordinate has the greatest attenuation of the calculation errors and improves the forecasting ability mostly. Li et al. [6] also designed a new COS coordinate, which effectively solved the problem of the poor real test results of COS coordinate in Li et al. [5]. The new COS coordinate is designed to be more flexible at the bottom. Li et al. [7] compared the effects of several SLEVE-hybrid coordinates on the simulation of typical cases, and found that the SLEVE-hybrid coordinates could effectively alleviate the problems of excessive south wind, false precipitation and incorrect weather systems in the GRAPES_Meso model. Zhang et al. [15] applied a new SLEVE-hybrid coordinate form proposed by Klemp [2] to the GRAPES_Meso model, which could smooth the terrain influence layer by layer with the height. The advantages of this coordinate were revealed through ideal tests and simple case simulation tests. Li et al.[8] applied this coordinate form to the high-resolution GRAPES-3km model, and conducted batch simulation experiments. The results showed that the average temperature and wind field simulation errors had been attenuated to a certain extent. The nationwide average precipitation ETS score also improved, and the score increase in the eastern part of the southwest region was more obvious.

The research on the SLEVE-hybrid coordinates mentioned above basically adopted two calculation schemes, which are defined as semi-analytical scheme and finite differential scheme respectively hereinafter. The calculation accuracy of the two schemes has some differences. Almost none research in this area has been carried out at present. This article will compare and analyze these differences from the dynamic core equation to the real simulation. The second section of the article mainly compares the difference of the theoretical and formula derivation of the two schemes. The third section performs a preliminary quantitative accuracy comparison for the two schemes. The fourth section carries out real simulations based on the dynamic core of the GRAPES regional model. Section 5 gives conclusions and discussions.

## 2 THEORETICAL AND FORMULA DERIVATION OF THE TWO SCHEMES

### 2.1 *Calculation differences between the two schemes*

The formula of SLEVE-hybrid coordinate (*Schär*et al. [10]) is $z = \hat{z} + \sum_{i=1}^{L} b_{h_i^*} \cdot Z_{S_i}(x,y)$, where $b_{h^*}$ is the terrain attenuation coefficient, and $L$ is the number of terrain scale spectrum (Li et al. [3]). After the coordinate deformation, the partial differentials of some model quantities have changed.

For the semi-analytical scheme, the main variables involved in the coordinate are the terrain slope $\phi_s$ ($\phi_{sx}$ and $\phi_{sy}$ are horizontal and vertical slope, respectively), $b_{h^*}$ and the coordinate vertical deformation variables $J_b$. The function form of $b_{h^*}$ is generally adopted in this scheme to ensure that at least the second-order derivation can be achieved, so that some physical quantities such as the divergence can obtain their analytical values. The horizontal partial differential deformation relationship is ($F$ represents any quantity),

$$\left(\frac{\partial F}{\partial x}\right)_z = \left(\frac{\partial F}{\partial x}\right)_{\hat{z}} + J_b \cdot \frac{\partial F}{\partial \hat{z}}\left(\sum_{i=1}^{L} b_{h_i^*} \cdot (\varphi_{sx})_i\right)_{\hat{z}}, \left(\frac{\partial F}{\partial y}\right)_z = \left(\frac{\partial F}{\partial y}\right)_{\hat{z}} + J_b \cdot \frac{\partial F}{\partial \hat{z}}\left(\sum_{i=1}^{L} b_{h_i^*} \cdot (\varphi_{sy})_i\right)_{\hat{z}};$$

The vertical partial differential deformation relations is $\frac{\partial F}{\partial z} = J_b \cdot \frac{\partial F}{\partial \hat{z}}$,

where

$$J_b = \frac{\partial \hat{z}}{\partial z} = \frac{1}{1 + \sum_1^L \frac{\partial\left(b_{h^*} \cdot zs\right)}{\partial \hat{z}}} = \frac{1}{1 + \sum_1^L \frac{\partial b_{h^*}}{\partial \hat{z}} \cdot zs}.$$

For the finite differential scheme, the quantities related to the coordinate mainly include the horizontal and vertical coordinate deformation variables $G_x$, $G_y$ and $G_z$, where $G_x = \frac{\partial z}{\partial x}$, $G_y = \frac{\partial z}{\partial y}$, and $G_z = \frac{\partial \hat{z}}{\partial z}$. Although we can gain their analytical values, their first-order derivative, used for the divergence calculation behind, can only be the finite differential values. This is where the main difference of the two scheme lies. The horizontal partial differential deformation relationship is

$$\frac{\partial F}{\partial x}\Big|_z = \frac{\partial F}{\partial x}\Big|_{\hat{z}} + G_Z \cdot G_x \cdot \frac{\partial F}{\partial \hat{z}},$$

$$\frac{\partial F}{\partial y}\Big|_z = \frac{\partial F}{\partial y}\Big|_{\hat{z}} + G_Z \cdot G_y \cdot \frac{\partial F}{\partial \hat{z}};$$

The vertical partial differential deformation relations is $\frac{\partial F}{\partial z} = G_Z \cdot \frac{\partial F}{\partial \hat{z}}$, where $G_z = J_b$.

2.2 *Comparison of relevant formulas of model dynamic core*

The influence of the terrain effect on the dynamic core through the coordinate is mainly reflected in PGF calculation, vertical velocity, divergence, etc. The following research compares and analyzes quantities such as vertical velocity and divergence of the two schemes.

For semi-analytical scheme:

(1) vertical velocity

$$w = \alpha_{w1} u + \alpha_{w2} v + \alpha_{w3} \hat{w},$$

where $w$ and $\hat{w}$ are vertical velocity before and after coordinate deformation; $u$ and $v$ are horizontal velocity.

$$\alpha_{w1} = b_{h^*} \cdot \phi_{sx},$$

$$\alpha_{w2} = b_{h^*} \cdot \phi_{sy},$$

$$\alpha_{w3} = 1 + \frac{\partial b_{h^*}}{\partial \hat{z}} \cdot Z_S(x, y),$$

and $J_b^{-1} = \frac{\partial z}{\partial \hat{z}} = 1 + \frac{\partial b_{h^*}}{\partial \hat{z}} \cdot Z_S(x, y) = \alpha_{w3}$.

(2) pressure gradient:

$$-Cp\theta\nabla_z\Pi = -Cp\theta\left(\nabla_{\hat{z}}\Pi + J_b \cdot b_{h^*} \cdot \frac{\partial \Pi}{\partial \hat{z}}\left(\phi_{sx} + \phi_{sy}\right)\right),$$

where $\theta$ is potential temperature, and $\Pi$ is Exner pressure.

(3) 3-D divergence:

$$D_3 = D_3\Big|_{\hat{z}} + J_b\left(\frac{db_{h^*}}{d\hat{z}} \cdot u \cdot \varphi_{Sx} + \frac{db_{h^*}}{d\hat{z}} \cdot v \cdot \varphi_{Sy} + \frac{\partial J_b^{-1}}{\partial \hat{z}}\hat{w}\right),$$

where $D_3\Big|_{\hat{z}} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \hat{w}}{\partial \hat{z}}\right)_z$.

For finite differential scheme, we get

(1) Vertical velocity,

$$w = G_x u + G_y v + \frac{\hat{w}}{G_z},$$

(2) pressure gradient,

$$-Cp\theta\nabla_z\Pi = -Cp\theta\left(\nabla_{\hat{z}}\Pi + G_z \cdot G_x \cdot \frac{\partial \Pi}{\partial \hat{z}} + G_z \cdot G_y \cdot \frac{\partial \Pi}{\partial \hat{z}}\right)$$

(3) 3-D divergence

$$D_3 = D_3\Big|_{\hat{z}} + G_z \cdot \left(\frac{dG_x}{d\hat{z}} \cdot u + \frac{dG_y}{d\hat{z}} \cdot v + \frac{\partial G_z^{-1}}{\partial \hat{z}}\hat{w}\right),$$

where $D_3\Big|_{\hat{z}} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \hat{w}}{\partial \hat{z}}\right)_z$.

There is $\frac{dG_x}{d\hat{z}}, \frac{dG_y}{d\hat{z}}$ and $\frac{\partial G_z^{-1}}{\partial \hat{z}}$ in the divergence calculation of the finite differential scheme, whose theoretical accuracy is lower than that of the semi-analytical scheme. Based on the above theoretical analysis and formula derivation, the two schemes are applied to the GRAPES model to compare their accuracy difference in the model simulation. This research is aimed to optimize the two schemes to provide a more accurate SLEVE-hybrid coordinate solution for operational application.

## 3 COMPARISON OF CALCULATION ACCURACY

According to the derivation of the previous section, we get $\frac{d(b_h \cdot \varphi_{sx})}{dz}\Big|_0^{Z_T} = \frac{dG_x}{dz}\Big|_0^{Z_T}$.

Within the integration area $[0, Z_T]$, the difference between the discrete integral value of $\frac{dG_x}{dz}$ and the analytical integral value of $\frac{d(b_h \cdot \varphi_{sx})}{dz}$ (Untch et al. [11]) can easily reflect the calculation accuracy of the two schemes.

For the SLEVE coordinate, we get,

$$\int_0^{Z_T}\left(\frac{d(b_h \cdot \varphi_{sx})}{dz}\right)dz = (b_h \cdot (z = Z_T) - b_h \cdot (z = 0)) \cdot \phi_{sx},$$

where $0 \leqslant \phi_{sx} < 1$.

In this way, we can easily gain the analytical value of $\int_0^{Z_T}\left(\frac{d(b_h \cdot \varphi_{sx})}{dz}\right)dz$.

Two sets of comparative experiments are designed to analyze the sensitivity of the discrete integral values of $\frac{dG_x}{dz}$ to the model layering. In the first set of experiments, the default setting of the 15 km resolution GRAPES_Meso model was selected for layering. While in the second set of experiments, the part with a larger thickness gradient in the higher levels was redesigned. Here, in order to make it easier to compare higher layers and test in the GRAPES model in the fourth section, both of the layering ensure that the height of the layer top remains unchanged. The distribution of the height (Fig. 1) and thickness (Fig. 2) of the model surfaces shows that the thickness of the redesigned high-level model surfaces changes more gently.

The calculation results show (Table 1) that the relative error (%) of the integration of $\frac{dG_x}{dz}$, with respect to the analytical solution, is slightly increased below the 27th layer and on the full model layers, but is greatly reduced from 96% to 13% above the 27th layer, which is in good consistence with the theoretical analysis. The larger the vertical gradient of the thickness of the model surface is, the larger the calculation error of the finite differential scheme is. If we do not plan to change the layering of the model, the semi-analytical scheme is more preferable.

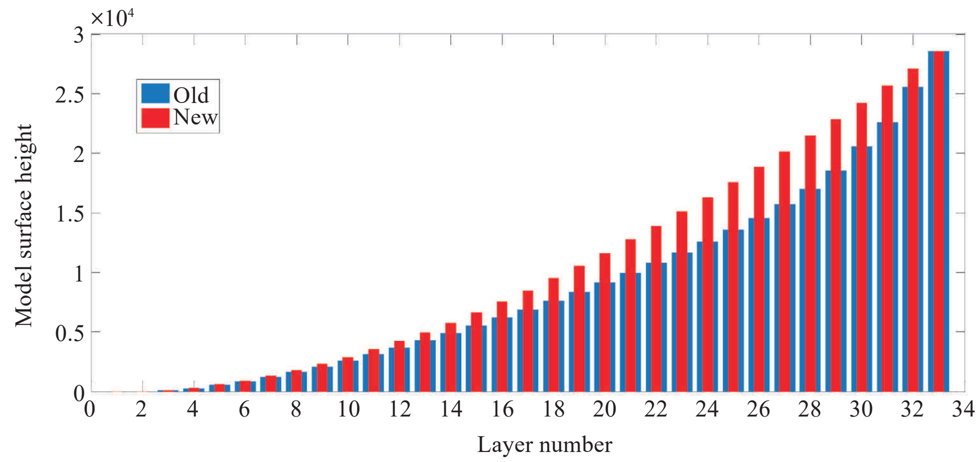


**Figure 1**. Height distribution of model surfaces. Blue bars for default model layers, and red bars for redesigned model layers.

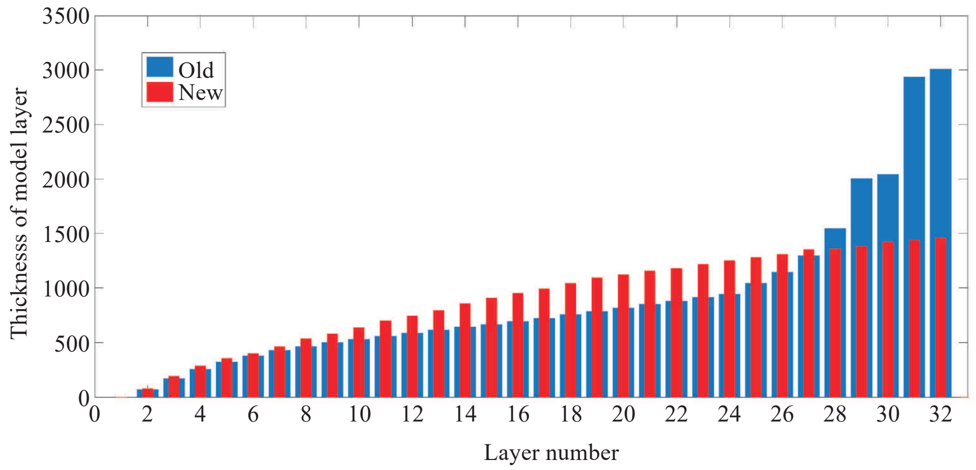


**Figure 2**. Thickness of model surfaces. Blue bars for default model layers, and red bars for redesigned model layers.

**Table 1**. Relative errors of $\frac{dG_x}{dz}$ under default model layers and redesigned model layers.

| | Relative error(%) | | |
|---|---|---|---|
| | 1-26L | 27-32L | 1-32L |
| Default model layers | 3 | 96 | 16 |
| Redesigned model layers | 5 | 13 | 18 |

## 4 BATCH TEST RESULTS WITH THE GRAPES_Meso DYNAMIC CORE

The following test examines and analyzes the performance of the two schemes in the long-term batch simulation, using the GRAPES_3km model to perform a 24-hour rolling forecast from September 1 to September 30, 2018, when the simulations are carried out under a weakly varying background. The horizontal resolution of the test is 0.15°×0.15°, the vertical layering is 31 layers, and the integration time step is 90s. The initial field and boundary field of the test are provided by the NCEP reanalysis data with a resolution of 1°×1° without data assimilation and the simulation range is 70°E−145°E, 10°N−60°N. In order to highlight the calculation difference of the dynamic core, all physical processes are closed, and one-dimensional reference profile is selected.

The two kinds of layering in Fig. 1 were used for the test setup. SLEVE1 coordinates (Li et al. [5]) were selected here, and four sets of comparative tests were conducted based on the semi-analytical scheme and the finite differential scheme, respectively. The four sets of comparative tests are: default layering with semi-analytical scheme, redesigned layering with semi-analytical scheme, default layering with finite differential scheme, and redesigned layering with finite differential scheme. The overall performance of the

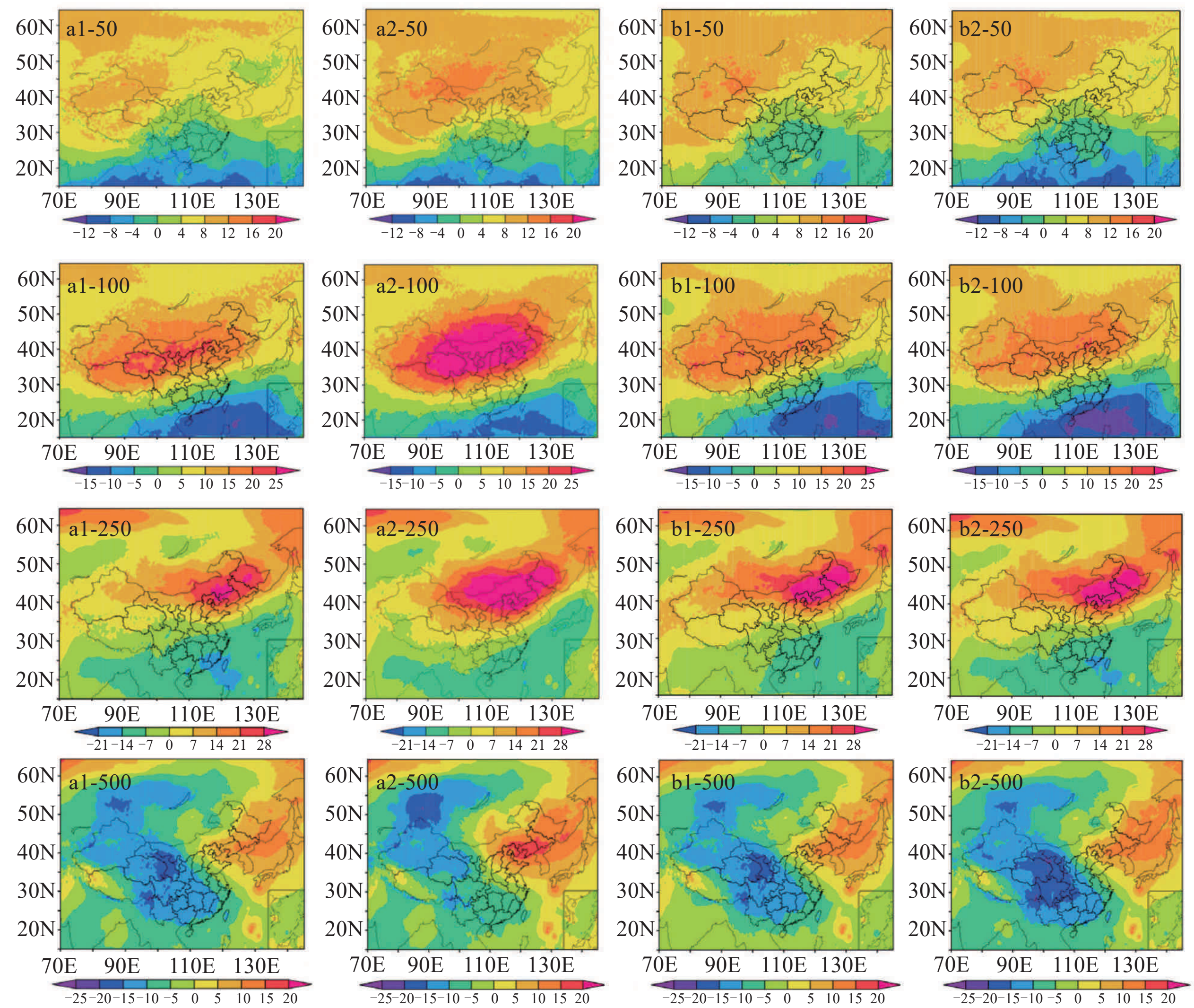


**Figure 3**. Monthly averaged 24 hrs forecast bias of geopotential height at 50hPa, 100hPa, 250hPa and 500hPa. (a1) Default layering of semi-analytical scheme, (a2) default layering of finite difference scheme, (b1) redesigned layering of semi-analytical scheme, and (b2) redesigned layering of finite difference scheme. Units: hPa.

model simulation does not show a consistent improvement with the redesigned layering. Especially, the average bias of the 50hPa temperature field (Fig. 4) becomes even larger. The model is a complex system, and there are many factors influencing this problem, but this does not affect the conclusion of the test in this article. The experimental results (Fig. 3 and Fig. 4) show that, with the default layering, the average bias at the model upper level of the finite differential scheme is more obvious than that of the semi-analytical scheme, and the height field is particularly obvious. However, the differences of the two schemes almost vanish with the redesigned layering. Therefore, the calculation error of the finite differential scheme will have a certain influence on the simulation of the dynamic core if the vertical gradient of the model surface thickness is not set properly. If we do not plan to change the layering of the model, the semi-analytical scheme is more preferable in the operational application.

## 5 SUMMARY

The calculation scheme of the SLEVE-hybrid coordinates in numerical forecasting model is not limited to one. This paper systematically introduces the differences between the semi-analytical scheme and the finite differential scheme dynamically and practically, and conclusions are drawn as follows:

(1) In the dynamic core simulation, the first-order finite difference of coordinate deformation variables is inevitable. The accuracy of the finite differential scheme is lower than that of the semi-analytical scheme in theory.

(2) The larger the vertical gradient of the layer thickness, the larger the relative errors of the first-order difference calculation of the coordinate deformation variables in the finite differential scheme.

(3) Although the simulation difference between the finite difference scheme and the semi-analytical scheme almost vanishes with a more uniform layering,

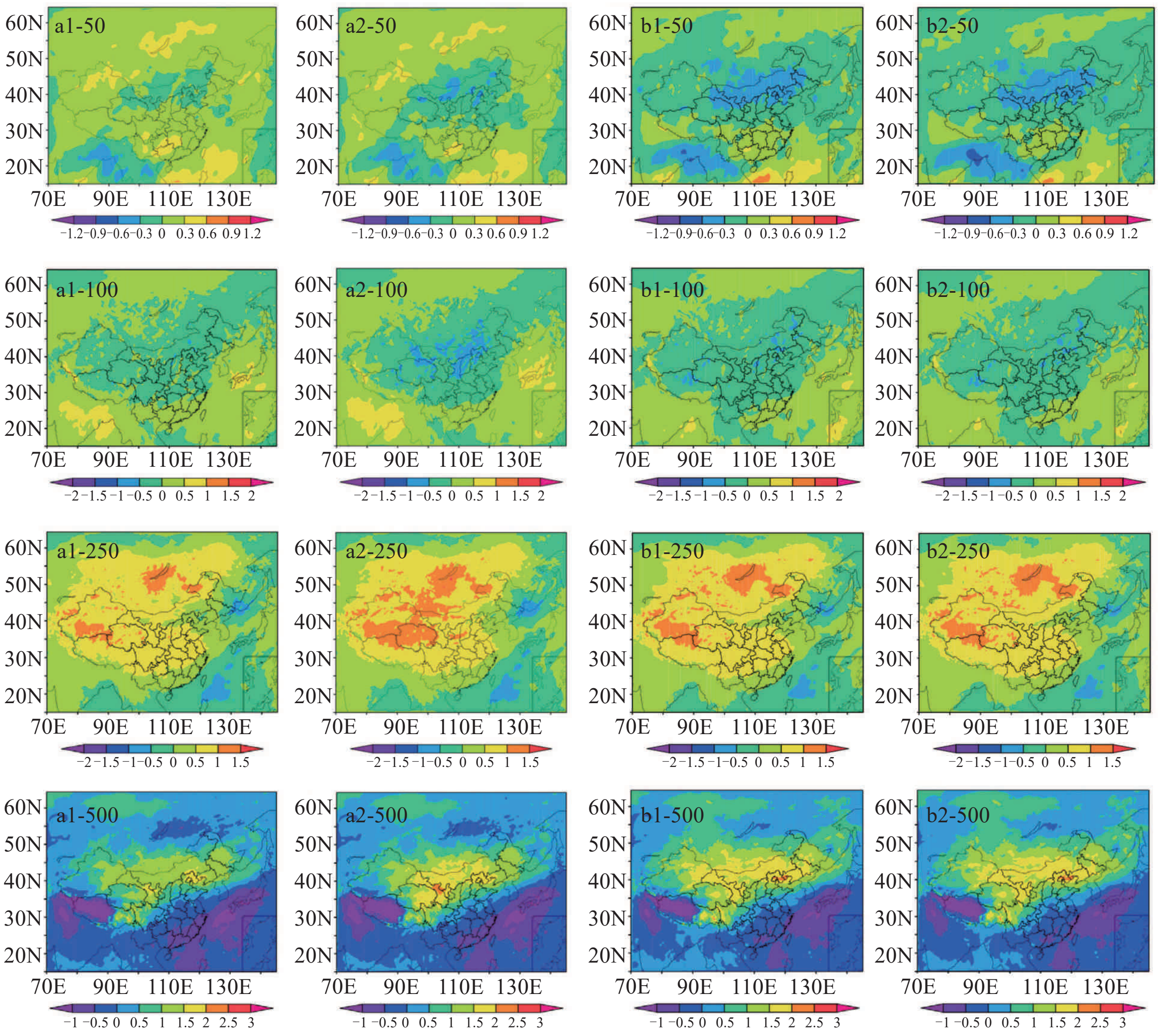


**Figure 4**. Monthly averaged 24 hrs forecast bias of temperature at 50hPa, 100hPa, 250hPa and 500hPa. (a1) Default layering of semi-analytical scheme, (a2) default layering of finite difference scheme, (b1) redesigned layering of semi-analytical scheme, and (b2) redesigned layering of finite difference scheme. Units: K.

the semi-analytical scheme will be more preferable for its good performance under the default layering.

The new coordinate proposed by Klemp [2] is different from the simple SLEVE1 coordinate selected here in the derivation of the dynamic equation. The comparison of the two schemes under the new coordinate needs further study. Moreover, the resolution of the GRAPES regional model is coarser in this article. Whether it can reach a consistent conclusion in the high-resolution GRAPES model also needs further research. Furthermore, the finite difference scheme is really efficient and concise to apply in the numerical models. Besides, there are some attempts that try to solve the vertical calculate problem of the finite difference scheme recently. Xu et al. [13] did some research on a second-order difference scheme for the non-uniformly distributed layers in GRAPES Model. Wu et al. [12] designed central difference scheme with second, fourth, and even sixth-order accuracy on non-uniform grids to solve the uniform layering problems of GRAPES model. These high order difference schemes should also be tried in the finite difference scheme for further research.

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